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Comparison of Eigenvalue Based Spectrum Sensing Techniques of Maximum-Minimum Eigenvalue and Energy-Minimum Eigenvalue with Energy Detection in Cognitive Radio

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Abstract-Uncertainties in spectrum sensing are the fundamental problems in cognitive radio. The prevailing effect of impulse, thermal and environmental noise on existing sensing techniques such as energy detection, matched filter and cyclostationary feature detections has necessitated the need for comparative studies of eigenvalues based sensing techniques in a noisy system. Two among the eigenvalue based spectrum sensing techniques, the Maximum Minimum Eigenvalue (MME) and the Energy with Minimum Eigenvalue (EME), have outstanding performances in the presence of noise as against the popular Energy Detection (ED) technique. In this work, the three techniques, the MME, EME and ED were applied to broadcasting frequency bands in a Rayleigh channel at low Signal to Noise Ratio (SNR), and their comparative performance was analysed. The algorithms for the three detection techniques were implemented in MATLAB, with simulated Primary User's (PU) signal in the VHF/UHF broadcasting frequency bands. The PU signal samples received together with noise by Secondary Users (SUs) was used to get the covariance matrix of the received signal. The difference between the statistical properties of the covariance matrix of the received signal and the noise signal gave a reliable detection test without requiring the knowledge of the PU parameters or the received noise variance. The ratio of the maximum and minimum eigenvalues was derived from the covariance matrix to determine the detection test statistics for MME, while the ratio of the received signal's energy with the minimum eigenvalue was used for EME detection test statistics. The detection tests were compared to the determined thresholds. The thresholds for the eigenvalue based techniques were determined using random matrix theories, that is, the probability of false alarm, number of samples and smoothing factor while the ED threshold was determined based on the received noise power. Using SNR values between 0 dB and -20 dB and multiple SU receivers, the result showed that the probability of detection for MME technique is 92% which is higher than the probability of detection for EME technique of 86%, while ED technique performs very poorly with the probability of detection of 1.8%. However, for large number of SU receivers, EME performed slightly better at very low SNR. Also, MME attains convergence faster than EME technique, as MME attained convergence at SNR of -6 dB but EME attained convergence at SNR of -2 dB.

Keywords- *Spectrum Sensing, Maximum-Minimum Eigenvalue, Energy with Minimum Eigenvalue, Energy Detection, Cognitive Radio, Primary User, Secondary User*

I. INTRODUCTION

Cognitive Radio (CR) was developed in order to combat the problem of radio frequency spectrum scarcity due to the fixed frequency spectrum allocation policy and increase in the use of wireless communication systems [1]. Though the electromagnetic radio frequency (RF) spectrum seems to be an inexhaustible and renewable natural resource, yet, it can be scarce if not well managed. The existing wireless communication systems and services with the development and deployment of new ones coupled with the fixed frequency spectrum allocation policy result in RF underutilization and at the same time scarcity of radio spectrum. Radio frequency ranges from 3 kHz – 3000 GHz have been valuable in communication industry [2], and as such, is needed to be well managed to avoid scarcity. The statistics of the current usage of the radio spectrum by several agencies revealed that some frequency bands are heavily used, some frequency bands are only partially occupied, while other frequency bands in the spectrum remain largely unoccupied and idle [3], [4].

A spectrum hole (also known as white space) is a band of frequencies assigned to a Primary User (PU), but not utilized by the user at a particular time and specific geographic location (vacant frequency band) [5]. PUs are those who hold the licensed channels while Secondary Users (SUs) are the unlicensed or opportunistic users [6].

Spectrum Sensing in CR is the process of monitoring a specific frequency band from time to time, in order to identify presence or absence of primary users. It can also be defined as the process of searching for unused frequency bands in the radio frequency spectrum [7]. It is the first aspects of cognitive radio cycle. Inaccurate spectrum sensing leads to imperfect and unreliable CR. It is therefore necessary to have reliable signal detection methods in Cognitive Radio Network (CRN).

Many studies have shown that local sensing techniques such as matched filter, cyclostationary based and energy detection are not often sufficient to provide accurate detection because they are primary user's (PU's) dependent [8]. An exception is the popular Energy Detection (ED) technique but which still performs poorly in a low Signal to Noise Ratio (SNR) environment [9]. Effect of fading, shadowing and other issues in the complicated wireless environment will affect the signal unreasonably [10].

Considering the uncertainties in spectrum sensing techniques in the presence of environmental noise, it is necessary to have a reliable spectrum sensing technique that will perform optimally for cognitive radio systems, where spectrum holes can be accurately detected and made available for secondary user without interference. In literature, various methods of spectrum sensing have performed reasonably well but not optimally [10], [11], [12].

A well performed detection technique that is effective for all scenarios, irrespective of the various environmental circumstances is the eigenvalue based spectrum sensing. Most of the existing sensing methods depend on the accuracy of the received PUs information for signal detection while eigenvalue based sensing do not. It detects without the knowledge of the primary users' parameters, such as the channel, modulation technique, the signal's power and so on.

Spectrum scarcity solution brought about by CR was proposed by Mitola and Maguire in 1999 [13] with an improvement of cognitive engine with the existing Software Defined Radio (SDR). Since the invention of CR paradigm, several spectrum sensing algorithms have been proposed with different signal detection performance. Reference [1] proposed an energy detection technique for spectrum sensing in cognitive radio, the algorithm performs well where the knowledge of the PUs' parameters was not known. It is easy, cheap, less sensing time but performs very poorly in a low SNR.

Reference [8] proposed maximum-minimum eigenvalue detection for CR based on covariance matrix of the received signals and Random Matrix Theories (RMT). The method retains the advantages of energy detection and overcomes the difficulty of noise uncertainty. Reference [14] proposed an improvement on spectrum sensing performance using cooperative eigenvectors. It is an improvement on the problem of noise uncertainties in the popular ED and can detect signal without the knowledge of the PUs' parameters. They made use of correlated signal and eigenvalues to prevent the effect of noise in a noisy signal environment. Cyclostationary feature detection technique was proposed by [15]. The algorithm performs better than the popular energy detection but requires large sensing time.

Reference [16] proposed the performance of cooperative eigenvalue spectrum sensing with a realistic receiver model under impulsive noise. It was deduced from its algorithm performance, that maximum minimum eigenvalue detection model improves better in the presence of impulse noise. The algorithm's classification accuracy outperforms a set of literature in comparison results. Reference [17] proposed a matched filter detection algorithm with a high signal detection ability in a low SNR but require the prior knowledge of the PU.

Eigenvalue based spectrum sensing technique has the functionality of high probability of detection and low probability of miss detection in a low SNR under Rayleigh channel. This is due to its non-dependence on the received signal's power. This makes it effective and invariably makes CR reliable. In the literature, the popular ED technique that has this functionality performs very poor in a low SNR [18]. The eigenvalue techniques that are known to perform well have been applied by researchers independently and separately to different noise channels, such as Rician, Nakagami etc. Nevertheless, the need to apply them to determine their comparative performances in low SNR Rayleigh channel in VHF/UHF broadcasting bands informs the motivation for this work.

In this work, the algorithms for Maximum Minimum Eigenvalue based spectrum sensing (MME), Energy with Minimum Eigenvalue based spectrum sensing (EME) and Energy Detection (ED) techniques in low SNR environment were implemented and simulated using randomly generated signals in VHF/UHF broadcasting bands under Rayleigh channel; and comparative performance evaluation of the spectrum sensing schemes using probability of detection was carried out. The ED technique was used as reference to consider the performances of the eigenvalue based techniques.

The rest of the paper is structured as follows; Section 2 gives mathematical foundation for the eigenvalue based detection methods. Section 3 explains the methodologies for MME, EME and ED. Simulation results based on randomly generated signals are given in Section 4, and conclusions are then drawn in Section 5.

II. MATHEMATICAL FOUNDATIONS FOR SPECTRUM DETECTION

A. Identification Problem

Detection of the received sample signal $x(n)$ can be reduced to an identification problem, with hypothesis test; H_o and H_1. H_o implies an absence of the signal, while H_1denotes presence of the signal.This is represented by Equation (1)

$$x_i(n) = \begin{cases} w_i(n) H_o \\ h_{ij} * s_j(n) + w_i(n) H_1 \end{cases} \quad (1)$$

where $x_i(n)$ is the received sample to be analysed, $w_i(n)$ is the Additive White Gaussian Noise (AWGN) with assumed zero mean and variance σ^2, h_{ij} is the complex channel response between the ith secondary user's receiver and the jth primary user's transmitter, i = 0, 1, 2, ….., L-1, where L is the total number of secondary users (receivers). $s_j(n)$ is the jth primary user's transmitted signal to be detected by the secondary user's receiver.

The goal is to observe the sample signal $x_i(n)$ using the three methods, the MME, EME and ED, decide the correct hypothesis based on the test statistics (T) being either greater or less than the threshold (γ). Characterizing the performance of such a decision rule is realized by using some sensing quality metrics which are probability of detection P_d, probability of false alarm P_{fa}, and probability of missed detection, $P_{m.}$ [19].

In this work, probability of false alarm P_{fa} is fixed to 0.1 while probability of detection P_d is varied. This was chosen because typically, P_{fa} is given a value between $10^{-1} – 10^{-2}$. The IEEE 802.22 standard recommends P_{fa} of 0.1 or less for spectrum sensing [20].

B. System Model for Eigenvalue Based Detection

Fig. 1 shows an illustration of one PU with k number of SUs. A time series sample matrix *x(n)* formed from each of the SUs is given in Equation (2)

$$x_i(n)= h_{ij} * s_j(n) + w_i(n) \quad (2)$$

where h_{ij} is the channel matrix that exists between the transmitter *j* of the PU to the receiver *i* at the SU (*i* = 1, 2, …, k) , $s_j(n)$ is the nth sample of the PU signal and $w_i(n)$is the additive noise at the cognitive radio receiver. For the CR spectrum sensing, a set of Ns number of observations or samples was obtained such that $s_j(n)$, for *n* = 0, 1, …, Ns – 1, will be used to decide if the primary signal is present or not. The noise component *w(n)* is modelled as an independent and identically distributed circularly symmetric, Gaussian noise with zero mean, that is independent of *s(n)* with unknown noise variance σ^2.

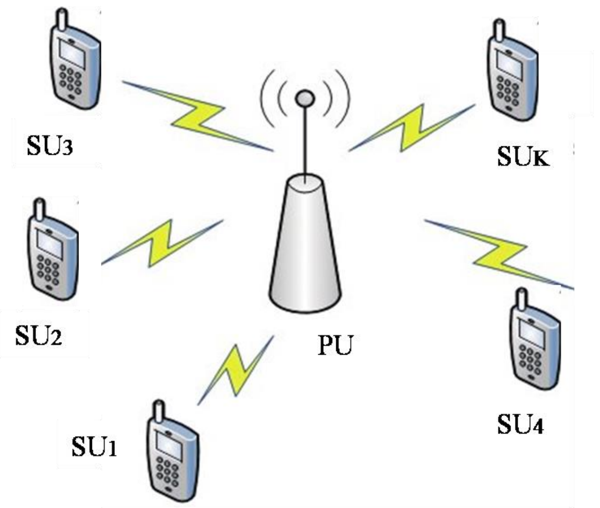

Figure 1. Representation of one PU with k number of SUs.

In this work, Double Side Band Suppressed Carrier (DSB-SC) modulation was used to obtain the sample signals from the PU. For the channel matrix *h*, Rayleigh channel was considered. A consecutive window of length L which is called the smoothing factor was considered. L models the number of receivers at the SU. Let P represents the number of source signal, which ranges from 1, 2, 3, …., L+1, T stands for the transpose of the received signal matrix, then the estimation of the received sequenced signals are given in equations (3) to (5) as:

$$x_i(n) = \left[x^T(n), x^T(n-1), x^T(n-L+1)\right]^T \quad (3)$$

$$w_i(n) = \left[w^T(n), w^T(n-1), w^T(n-L+1)\right]^T \quad (4)$$

$$s_j(n) = \begin{bmatrix} s_1(n), s_1(n-1),., s_1(n-N_1-L+1),., \\ s_P(n), s_P(n-1),., s_P(n-N_P-L+1) \end{bmatrix}^T \quad (5)$$

In the eigenvalue based method, the eigenvalues of the covariance matrix of the received signal at each SU are used. The received signals' covariance matrix of the samples is given as [17]:

$$C_x(N_S)= \frac{1}{N_S}\sum_{n-L-1}^{L-2+N_S} x(n)x(n)^\dagger \quad (6)$$

where C_x is the covariance matrix, (N_S) is number of samples collected, $x(n)$ is estimated received sample signal, $x(n)^\dagger$ is the transpose conjugate of the estimated signal sample and L is the number of consecutive outputs called smoothing factor.

The obtained eigenvalues of the covariance matrix of the sample signals serves as the test statistics that is compared with the threshold as illustrated in Fig. 2.

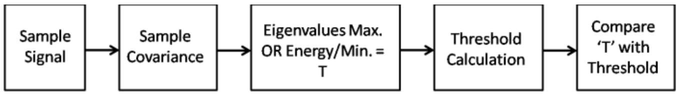

Figure 2. Detection principle of eigenvalue methods [17].

Based on RMT, decision threshold expression has been derived. This method overcomes problematic noise uncertainty encountered by energy detectors. Moreover, correlation among signal samples has been incorporated by covariance matrix. Two forms of eigenvalue estimation techniques are considered in this work – the Maximum-Minimum Eigenvalue (MME) detection technique and the Energy with Minimum Eigenvalue (EME) detection technique. The methodologies of the two eigenvalue based detection and the energy detection methods are discussed in the next section.

III. METHODOLOGIES FOR MME, EME AND ED DETECTION

A. Maximum-Minimum Eigenvalue (MME) Detection

MME detection compares the threshold with the ratio of the maximum eigenvalue to the minimum eigenvalue. The threshold is not based on noise power and it is estimated by using the number of samples, smoothing factor and probability of false alarm P_{fa}, in the following steps:

Step 1: Compute the sample Covariance Matrix of the received signal

The sample covariance matrix of the received signal $x(n)$ was computed using Equation (6).

Step 2: Calculate the Maximum and Minimum Eigenvalues of the sample Covariance Matrix.

Maximum (λ_{max}) and minimum (λ_{min}) eigenvalues and their corresponding eigenvectors were calculated from the received signal covariance matrix using Rayleigh quotient formula given in Equation (7)

$$\lambda_{max} = \frac{C_x(N_S)\, x.x}{x.x} \quad (7)$$

To obtain the minimum (λ_{min}) eigenvalue, Equation (8) is used

$$\lambda_{min} = \frac{tr(C_x(N_S)) - \lambda_{max}}{M-1} \quad (8)$$

where tr(.) is the trace of a matrix, and M is the length of the covariance matrix $C_x(N_S)$

Step 3: Compute the Probability of False Alarm (P_{faMME})

Based on the work of [9], equations (9) to (11) were used to compute the probability of false alarm for MME

$$P_{faMME} = P\left(\frac{\lambda_{max}(C_x(N_s)) - \mu}{V} > \frac{\gamma_{MME}\left(\sqrt{C_x(N_s)} - \sqrt{ML}\right)^2 - \mu}{V}\right) \quad (9)$$

where

$$\mu = \left(\sqrt{N_S - 1} + \sqrt{ML}\right)^2 \quad (10)$$

and

$$V = \left(\sqrt{N_S - 1} + \sqrt{ML}\right)\left(\frac{1}{\sqrt{N_S - 1}} + \frac{1}{\sqrt{ML}}\right)^{1/3} \quad (11)$$

To study the eigenvalue distribution of a random matrix, the joint probability density function (PDF) of ordered eigenvalues of a Wishart random matrix is used. The expression of the PDF is very complicated. To address this problem, the Tracy-Widom function distribution $F_1(.)$ is used to obtain P_{faMME} as shown in Equation (12)

$$P_{faMME} = 1 - F_1\left(\frac{\gamma_{MME}\left(\sqrt{N_s} - \sqrt{ML}\right)^2 - \mu}{V}\right) \quad (12)$$

where L is the “smoothing factor” that serves as the window length of the received sample signal. L is also used for the number of receivers used in this work, N_S is the number of sample signals at the receivers, and $F_1(.)$ is the Tracy-Widom distribution. Equation (12) is the probability of false alarm (P_{faMME}) obtained when using the MME detector.

Step 4: Compute the Threshold.

From Equation (12), the following equations for the MME threshold (γ_{MME}) was derived [9]:

$$F_1\left(\frac{\gamma_{MME}\left(\sqrt{N_s} - \sqrt{ML}\right)^2 - \mu}{V}\right) = 1 - P_{faMME} \quad (13)$$

Equivalently,

$$\gamma_{MME}\left(\frac{\left(\sqrt{N_s} - \sqrt{ML}\right)^2 - \mu}{V}\right) = F_1^{-1}\left(1 - P_{faMME}\right) \quad (14)$$

Substituting μ and V into Equation (14), the threshold γ_{MME} was obtained as in Equation (15)

$$\gamma_{MME} = \frac{\left(\sqrt{N_s} + \sqrt{ML}\right)^2}{\left(\sqrt{N_s} - \sqrt{ML}\right)^2}\left(1 + \frac{\left(\sqrt{N_s} + \sqrt{ML}\right)^{-2/3}}{\left(N_s ML\right)^{1/6}} F_1^{-1}\left(1 - P_{faMME}\right)\right) \quad (15)$$

where γ_{MME} is the threshold for MME based on the P_{faMME} and F_1^{-1} (.) is the inverse of the Tracy-Widom distribution function.

From Equation (15), it was noted that γ_{MME} is not related to noise power unlike the energy detection (ED) method. γ_{MME} is computed based on M, N_s, L, and P_{faMME} irrespective of the received signal and noise.

Step 5: Probability of Detection (p_{dMME})

The equation of the probability of detection for MME p_{dMME} is expressed in Equation (16) [9]

$$p_{dMME} = 1 - F_1\left(\frac{\gamma_{MME}N_s + N_s\left(\gamma_{MME}\,\lambda_{max} - \lambda_{min}\right)/\sigma_w^2 - \mu}{V}\right) \quad (16)$$

From Equation (16), the P_{dMME} is related to the number of samples N_S, the threshold of MME γ_{MME}, μ and V which retains their definitions, and then the maximum λ_{max} and the minimum λ_{min} eigenvalues of the signal covariance matrix.

Step 6: Apply the Test Statistic Decision Rule

For MME, the test statistics T_{MME} is the ratio of the maximum eigenvalue to the minimum eigenvalue of the sampled covariance matrix of the received signals. This is given in Equation (17).

$$T_{MME} = \frac{\lambda_{max}}{\lambda_{min}} \quad (17)$$

The decision rule is given in Equation (18)

$$Decision\ Rule = \begin{cases} \text{If } T_{MME} \geq \gamma_{MME} \text{ then } H_1 \text{ i.e., the PU is present.} \\ \text{If } T_{MME} < \gamma_{MME} \text{ then } H_o \text{ i.e., the PU is absent.} \end{cases} \quad (18)$$

where γ_{MME} is obtained from Equation (15) and T_{MME} is obtained from Equation (17).

B. Energy with Minimum Eigenvalue (EME) Detection

EME detection technique compares the threshold with the value of the ratio of the average signal power to the minimum eigenvalue. Just like MME and unlike ED, the threshold is not based on noise power. The following steps were followed to compute the EME.

Step 1: Compute the sample Covariance Matrix of the received signal

The sample covariance matrix of the received signal $x(n)$ was computed using equation (6) just like it was computed for MME.

Step 2: Calculate the average power of the received signal.

Let E(N_S) be the average energy of the received signal [9], then

$$E(N_S) = \frac{1}{MN_S}\sum_{i=1}^{M}\sum_{n=0}^{N_S-1}|x_i(n)|^2 \quad (19)$$

where $x_i(n)$ is the received signal. Since N_s is large, the central limit theorem shows that $E(N_S)$ can be approximated by the Gaussian distribution with mean σ_w^2 and variance $\frac{2\sigma_w^4}{MN_S}$.

Step 3: Compute the minimum eigenvalue of the received covariance matrix.

This is done as discussed in Step 2 of MME.

Step 4: Compute the Probability of False Alarm (P_{faEME}) for EME

The P_{faEME} is as expressed in equations (20) and (21).

$$P_{faEME} = \left[\frac{E(N_S)-\sigma_w^2}{\sqrt{\frac{2}{MN_S\sigma_w^2}}} > \frac{\gamma_{EME}\sqrt{M}\left(\sqrt{N_S}-\sqrt{ML}\right)^2-\sqrt{MN_S}}{\sqrt{2N_S}}\right] \quad (20)$$

$$P_{faEME} = Q\left[\frac{\gamma_{EME}\sqrt{M}\left(\sqrt{N_S}-\sqrt{ML}\right)^2-\sqrt{MN_S}}{\sqrt{2N_S}}\right] \quad (21)$$

where

$$Q(t) = \frac{1}{\sqrt{2\pi}}\int_t^{+\infty} e^{-\frac{u^2}{2}}\,du \quad (22)$$

Q(t) is called the Q-function which is a convenient way to express right-tail probabilities for Gaussian random variables. Q(t) is the probability that a standard normal random variable (zero mean, unit variance) exceeds t.

Step 5: Compute the Threshold (γ_{EME}) for EME

From equations (21) and (22), Equation (23) is derived for threshold (γ_{EME}) [9].

$$\gamma_{EME} = \left(\sqrt{\frac{2}{MN_S}}Q^{-1}(P_{faEME}) + 1\right)\frac{N_S}{\left(\sqrt{N_S}-\sqrt{ML}\right)^2} \quad (23)$$

From Equation (22), it can be observed that for EME, the threshold γ_{EME} is not related to noise power. The threshold γ_{EME} is computed based only on N_S, L and P_{faEME} irrespective of the signal and noise power.

Step 6: Compute the Probability of Detection (p_{dEME}) for EME

The Energy $E(N_S)$ of the signal sample is given below [9]:

$$E(N_S) = \frac{Tr(C_x(N_S))}{ML} \quad (24)$$

$$E(N_S) \approx \frac{Tr(HC_SH^T)}{ML} + \frac{Tr(C_w(N_S))}{ML} \quad (25)$$

where Tr (.) means the trace of a matrix.

With further analysis, the approximate equation for P_{dEME} of EME is given in Equation (26)

$$P_{dEME} = Q\left(\frac{\gamma_{EME}\left(\rho_{ML}+\frac{\sigma_w^2}{N_S}\left(\sqrt{N_S}-\sqrt{ML}\right)\right)-\frac{Tr(HR_SH^T)}{ML}-\sigma_w^2}{\sqrt{\frac{2}{MN_S}}\sigma_w^2}\right) \quad (26)$$

Equation (26) gives the equation for the probability of detection for EME. From this formula, the P_{dEME} is related to the numbers of samples N_S and the average and minimum eigenvalues for the signal covariance matrix.

Step 7: Apply the test statistic decision rule.

For EME, the test statistics T_{EME} is the ratio of the average energy of the received signal to the minimum eigenvalue of the covariance matrix of the received signal. This is given in Equation (27).

$$T_{EME} = \frac{E(N_S)}{\lambda_{min}} \quad (27)$$

The decision rule becomes

$$Decision\ Rule = \begin{cases} \text{If } T_{EME} > \gamma_{EME} \text{ then } H_1 \text{ i.e., the PU is present.} \\ \text{If } T_{EME} < \gamma_{EME} \text{ then } H_o \text{ i.e., the PU is absent.} \end{cases} \quad (28)$$

where γ_{EME} is obtained from Equation (23) and T_{EME} is obtained from Equation (27).

C. Energy Detection (ED) Method

The signal is filtered before the energy detection is implemented. Let *f (l) (l* = 0, 1, . . ., *L)* be a filter or the combination of bank of filters. The received signal after filtering is given in Equation (29) and (30) [21] as:

$$y_i(n) = \sum_{l=0}^{L} f(l)x_i(n-1) \quad (29)$$

The energy detector after the filtering is therefore

$$T_{ED,Filter} = \frac{1}{N}\sum_{n=1}^{N}|y(n)|^2 \quad (30)$$

The output signal T_{ED} is compared to the threshold γ in order to decide whether a signal is present in the frequency band or not as shown in Equation (31).

$$T_{ED} \gtrless \gamma \quad (31)$$

Energy detection can also be done in frequency domain [1].

Since there is no prior knowledge about PU signal required, hence it is a non-coherent scheme. It is the most popular and simplest detector with less computational and implementation complexity, less delay relative to other methods. Under low SNR or noise uncertainties, its performance degrades greatly. If noise uncertainty exists, this method is poor. However, the method requires the accurate knowledge of noise power and hence is susceptible to noise uncertainty [21].

The following steps are followed to compute the algorithm for the energy detection method

Step 1: Compute the energy of the received signal

$$S_{i,}(k) = \frac{1}{P}\sum_{p=1}^{P}\left|X_{i,p}(k)\right|^2 \quad (32)$$

where $S_i(k)$ is the Power Spectral Density (PSD) of the received signal $x_i(n)$.

Step 2: Compute the Probability of Detection (P_{dED})

$$P_{dED} = Q\left[\frac{\gamma_{ED} - N\left(\left(W_n^2 + S_n^2\right)\right)}{\sqrt{2N\left(\left(W_n^2 + S_n^2\right)\right)^2}}\right] \quad (33)$$

Step 3: Compute the Probability of False Alarm P_{faED}

$$P_{faED} = Q\left[\frac{\gamma_{ED} - NW_n^2}{\sqrt{2NW_n^4}}\right] \quad (34)$$

Step 4: Compute the Threshold

$$\gamma = \left(Q^{-1}\left(P_{fd}\right)\sqrt{2N} + N\right)w_n^2 \quad (35)$$

Step 5: Apply the test statistic decision rule.

The test statistics T_{ED} for the ED method is given as Equation (36)

$$T_{ED} = x(n)^2 \quad (36)$$

The decision rule is as expressed in Equation (37).

$$Decision\ Rule = \begin{cases} \text{If } T_{ED} > \gamma_{ED} \text{ then } H_1 \text{ i.e., the PU is present.} \\ \text{If } T_{ED} < \gamma_{ED} \text{ then } H_o \text{ i.e., the PU is absent.} \end{cases} \quad (37)$$

where γ_{ED} is obtained from Equation (35) and T_{ED} is obtained from Equation (36)

Energy detection is easy to implement and does not require any prior knowledge about the PU signal, which makes it one of the most used technique. However, it is very sensitive to the noise and cannot distinguish between the signal and the noise when the signal power is low, that is, it degrades in performance at low SNRs. This is the reason why in this work, the performances of the MME and EME with respect to the ED in Rayleigh channel were compared. Fig. 3 shows the flow chart of how the comparison of the probability of detection was achieved between the ED, the MME and the EME methods.

IV. RESULTS AND DISCUSSION

This Section presents the receiver operating characteristics (ROC) curve simulation results obtained when randomly generated signals were used to evaluate the performance of the three chosen detection methods (MME, EME and ED) for the presence of a PU in a CRN.

A. *Performance Evaluation of MME and ED Methods*

Fig. 4 shows the ROC curve for the performance of MME compared with ED for different number of receivers, L, under different range of low SNRs in Rayleigh channel.

In order to aid the discussion and analysis, some important terms and parameters were created. These are described as follows.

(i) For this work, a low range of SNR from -20dB to 0dB was used to represent the worst case scenario of low SNRs in the CRN. This range is called $LSNR_{range}$, which means low SNR Range is from -20dB to 0dB. In situations where it was necessary to create a sub-range low SNR out of $LSNR_{range}$, such instances were specified.

(ii) For this work, specific ranges of even numbers of receivers, L were used. This range is referred to as $LREC_{range}$ which means range of L receivers for L of 2, 4, 6, 8 and 10 number of receivers. In situations where it was necessary to specify a particular L or create a sub-range out of $LREC_{range}$, such instances were specified.

(iii) The probability of detection (P_d) was divided into different regions for categorization. Each category was also coded.

Fig. 5 shows how the P_d of each region of Fig. 4 was categorized and coded. Table 1 shows the P_d regions, category and the code associated with each category.

1) *Creation of P_d Sample Size for Analysis*

From Fig. 4, P_d at SNR of -20dB, -18dB, -16dB, -14dB, -12dB, -10dB, -8dB, -6dB, -4dB, -2dB, and 0dB for $LREC_{range}$ were chosen to form the sample size for analysis. Table 2 shows the generated sample size.

2) *Distribution of P_d in the Regions from the Sample Size of MME vs ED*

From Fig. 4 and Fig. 5, the number of points that falls within each P_d region was determined from which was calculated the percentage distribution of the P_d points in each P_d region for $LREC_{range}$. Table 2 shows the distribution of the number of P_d points and the average percentage composition of the P_d points in each region based on the sample size.

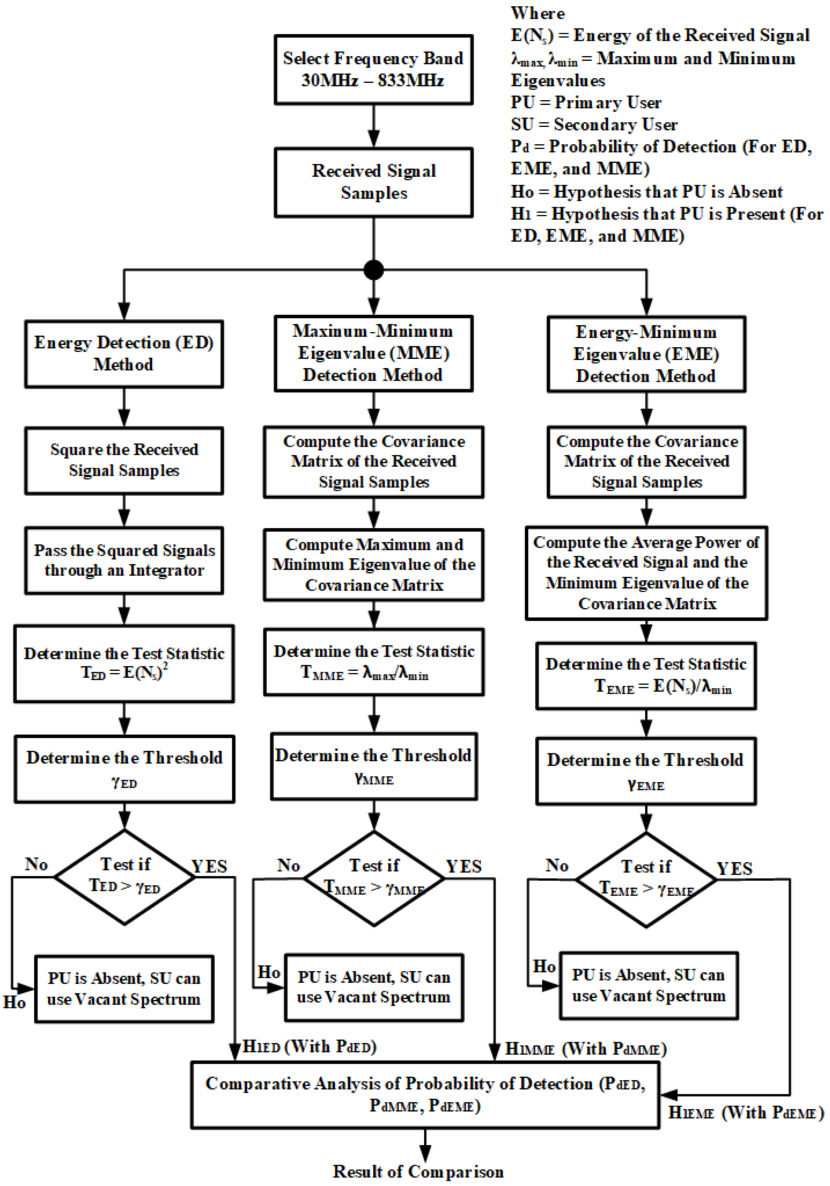

Figure 3. Comparison flow chart of the probability of detection between the ED method, the MME and the EME methods.

TABLE I. P_D REGIONS, CATEGORY AND CODE

P_d Regions	Category	Code
0.9000 – 0.9999	Excellent	EXR
0.8000 – 0.8999	Very Good	VGR
0.7000 – 0.7999	Upper Good	UGR
0.6000 – 0.6999	Lower Good	LGR
0.5000 – 0.5999	Fair	FIR
0.4000 – 0.4999	Upper Poor	UPR
0.3000 – 0.3999	Lower Poor	LPR
0.2000 – 0.2999	Very Poor	VPR
0.1000 – 0.1999	Insignificant	ISR

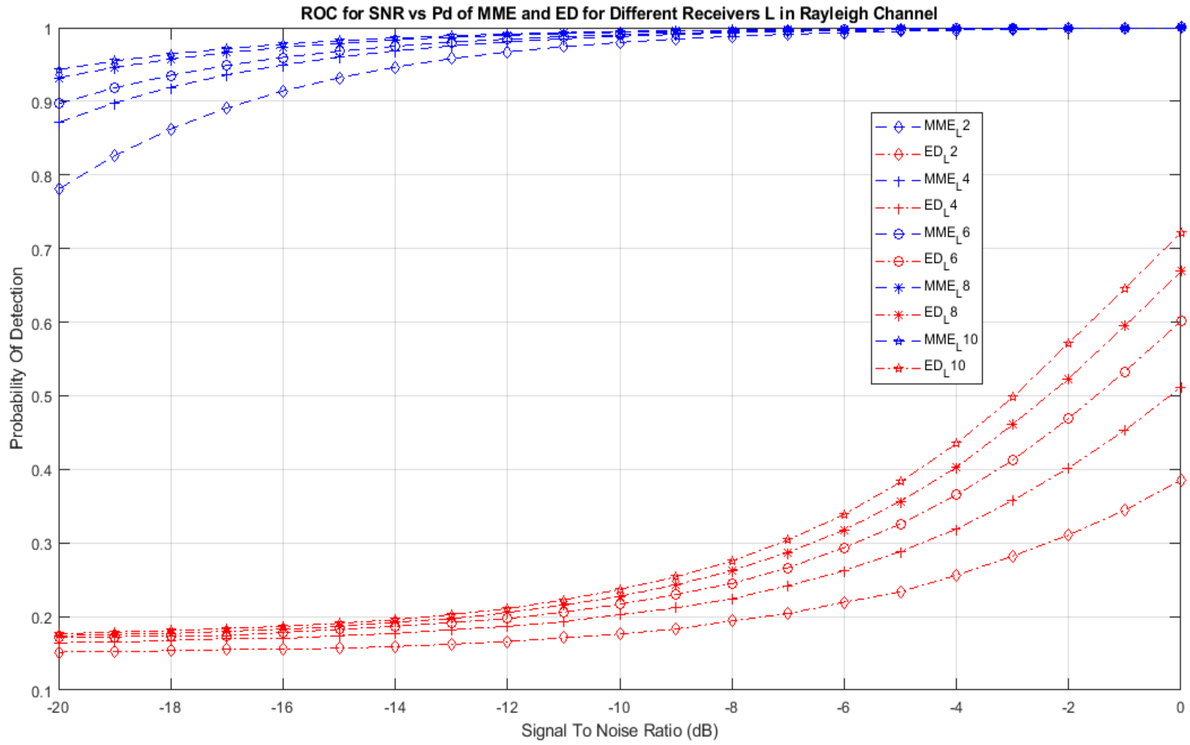

Figure 4. ROC for the performance of MME (|Blue) and ED (Red) for different receivers L.

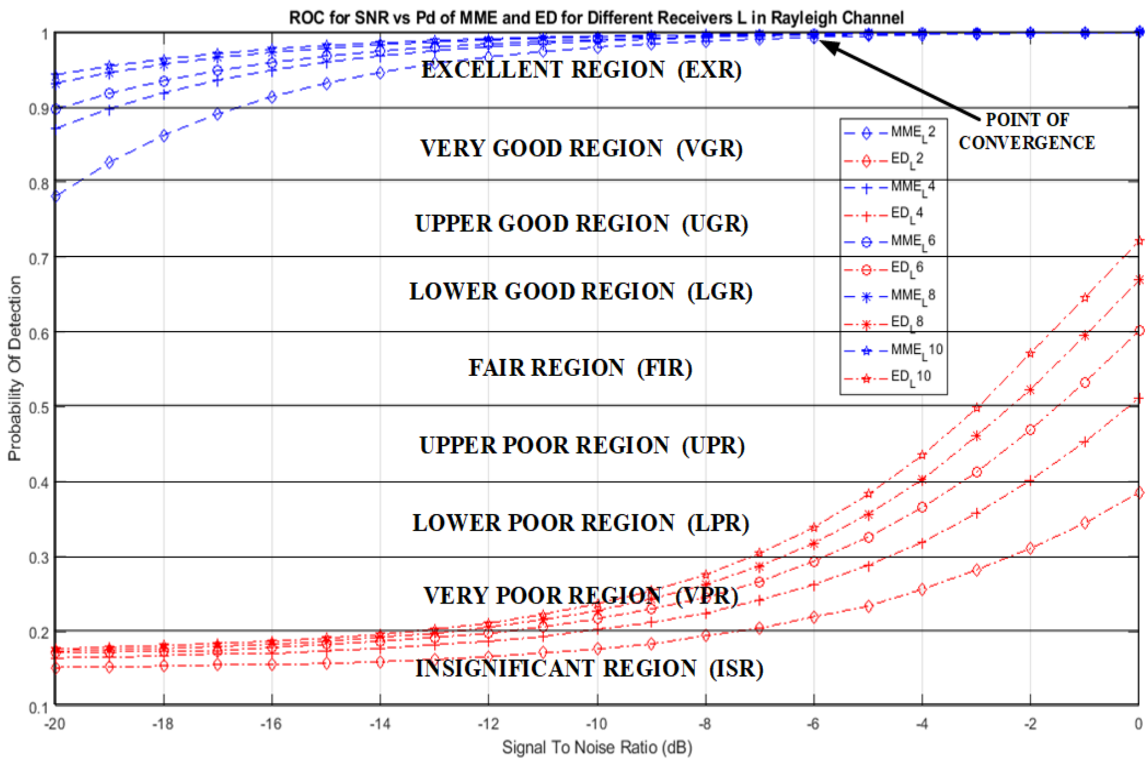

Figure 5. P_d Categorization and coding of each region of Figure 4.

TABLE II. DISTRIBUTION OF P_D POINTS IN EACH REGION FOR ED AND MME

P_d Region	ED		MME	
	P_d (Points)	P_d (%)	P_d (Points)	P_d (%)
EXR	-	-	51	92.7
VGR	-	-	3	5.5
UGR	1	1.8	1	1.8
LGR	2	3.6	-	-
FIR	4	7.3	-	-
UPR	4	7.3	-	-
LPR	6	10.9	-	-
VPR	13	23.6	-	-
ISR	25	45.5	-	-

3) Observations and Discussions

Based on the distribution of P_d points in the regions shown in Table 2, Fig. 4 and Fig. 5, the following observations were extracted and their implications are described thus:

(1) The P_d of MME for $LREC_{range}$ and $LSNR_{range}$ falls within the region of UGR up to EXR with 92.7% of the P_d points in the EXR region, 5.5% falls in the VGR region and only 1.8% falls in the UGR region.

(2) The P_d of ED for $LREC_{range}$ and $LSNR_{range}$ falls within the region of UGR down to ISR region with 45% of the P_d contained in the ISR region, the VPR region has 23.6%, LPR region has 10.9%, UPR and FIR both contain 7.3%, the LGR has 3.6% and the UGR has a very low percentage of 1.8%.

The implication of the observations made in (1) and (2) is that for low SNR scenario, the MME out-performs the ED, hence the MME method is a more reliable detection method at low SNRs than the ED.

(3) From the ROC curves in Fig. 4 and Fig. 5, it was observed that for SNR_{range} from -20dB to -14dB, the ROC for ED lies in the ISR region for all $LREC_{range.}$ The implication of this is that at this SNR_{range} of -20dB to -14dB, the P_d of the ED method is less than 0.2, hence, all the detection made by the receivers using the ED method of detection at this range is insignificant.

(4) From Fig. 4 and Fig. 5 for multiple SUs, the MME detection method attained a convergence at SNR of -6dB. Also, starting from -6dB up to 0dB, the MME method showed P_d of 0.9999 for all $LREC_{range}$. The implication of this observation is that starting from -6dB to 0dB the detection made by the receivers using the MME detection method is extremely reliable.

B. Performance Evaluation of EME and ED Methods

Fig. 6 shows the ROC for the performance of EME compared with ED for $LSNR_{range}$ and $LREC_{range}$. The categorization made in Table 1 was also applied for Fig. 6. The same procedure used for developing the sample size for the MME versus ED was also adopted for the EME versus ED.

1) Distribution of P_d in the Regions from the Sample Size of EME vs ED

From Fig. 6, the number of points that falls within each P_d region was determined from which was calculated the percentage distribution of the P_d points in each P_d region for $LREC_{range}$. Table 3 shows the distribution analysis for EME and ED.

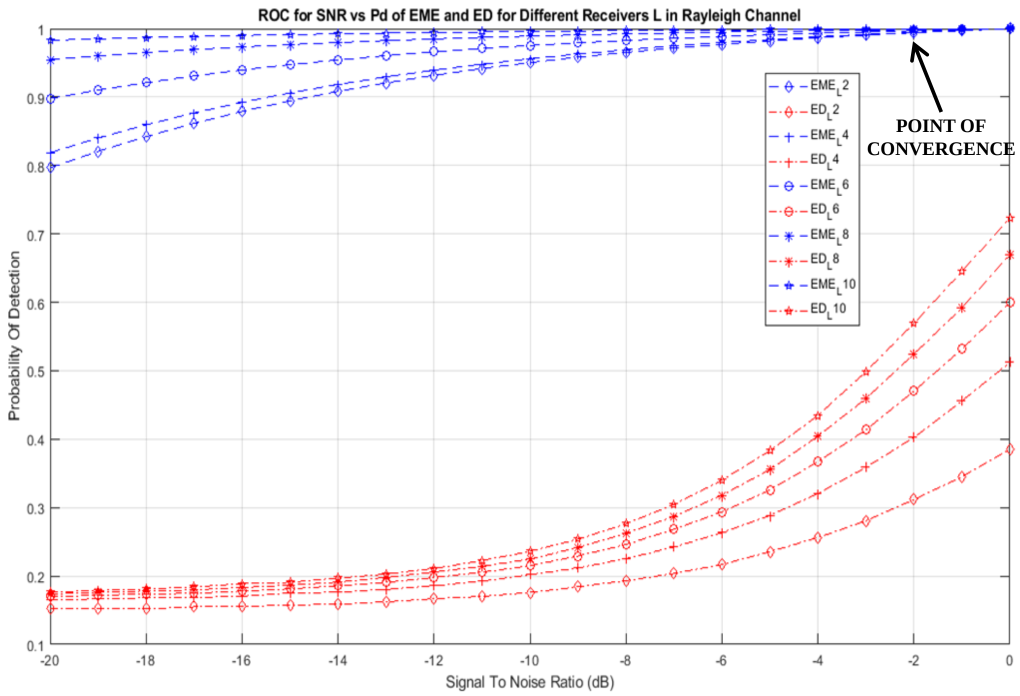

Figure 6. ROC for the Performance of EME (|Blue) and ED (Red) for different receivers L.

TABLE III. DISTRIBUTION OF P_D POINTS IN EACH REGION FOR ED AND EME

P_d Region	ED		EME	
	P_d (Points)	P_d (%)	P_d (Points)	P_d (%)
EXR	-	-	48	87.3
VGR	-	-	6	10.9
UGR	1	1.8	1	1.8
LGR	2	3.6	-	-
FIR	4	7.3	-	-
UPR	4	7.3	-	-
LPR	6	10.9	-	-
VPR	13	23.6	-	-
ISR	25	45.5	-	-

2) *Observations and Discussions*

Based on the distribution of P_d points in the regions shown in Table 3, as well as Fig. 6, the following observations were extracted and their implications are discussed thus:

(1) The P_d of EME for $LREC_{range}$ and $LSNR_{range}$ falls within the region of UGR up to EXR with 87.3% of the P_d points in the EXR region, 10.9% falls in the VGR region and only 1.8% falls in the UGR region.

(2) The P_d of ED for $LREC_{range}$ and $LSNR_{range}$ falls within the region of UGR down to ISR region with 45% of the P_d contained in the ISR region, the VPR region has 23.6%, LPR region has 10.9%, UPR and FIR both contain 7.3%, the LGR has 3.6% and the UGR has a very low percentage of 1.8%. The implication of the observations made in (1) and (2) is that for low SNR scenario, the EME performs better than the ED, hence the EME method is a more reliable detection method at low SNRs than the ED.

(3) From the ROC curves in Fig. 6, it was observed that for SNR_{range} from -20dB to -14dB, the ROC for ED lies in the ISR region for all $LREC_{range.}$ The implication of this is that at this SNR_{range} of -20dB to -14dB, the P_d of the ED method is less than 0.2, hence, all the detection made by the receivers using the ED method of detection at this range is insignificant.

(4) From Fig. 6 for multiple SUs, the EME detection method attained a convergence at SNR of -2dB. Also, starting from -2dB up to 0dB, the EME method showed P_d of 0.9999 for all $LREC_{range}$. The implication of this observation is that starting from -2dB to 0dB the detection made by the receivers using the EME detection method is extremely reliable.

C. *Performance Comparison of MME and EME*

In previous sections, it was shown that both MME and EME out-perform the ED at $LSNR_{range}$ and $LREC_{range}$ in Rayleigh channel for the specified CR broadcasting frequency bands. In this Section, the performances of MME and EME for detection are compared.

Fig. 7 shows the ROC plots for MME and EME. Table 4 shows the percentage distribution of P_d points in all regions for ED, MME and EME.

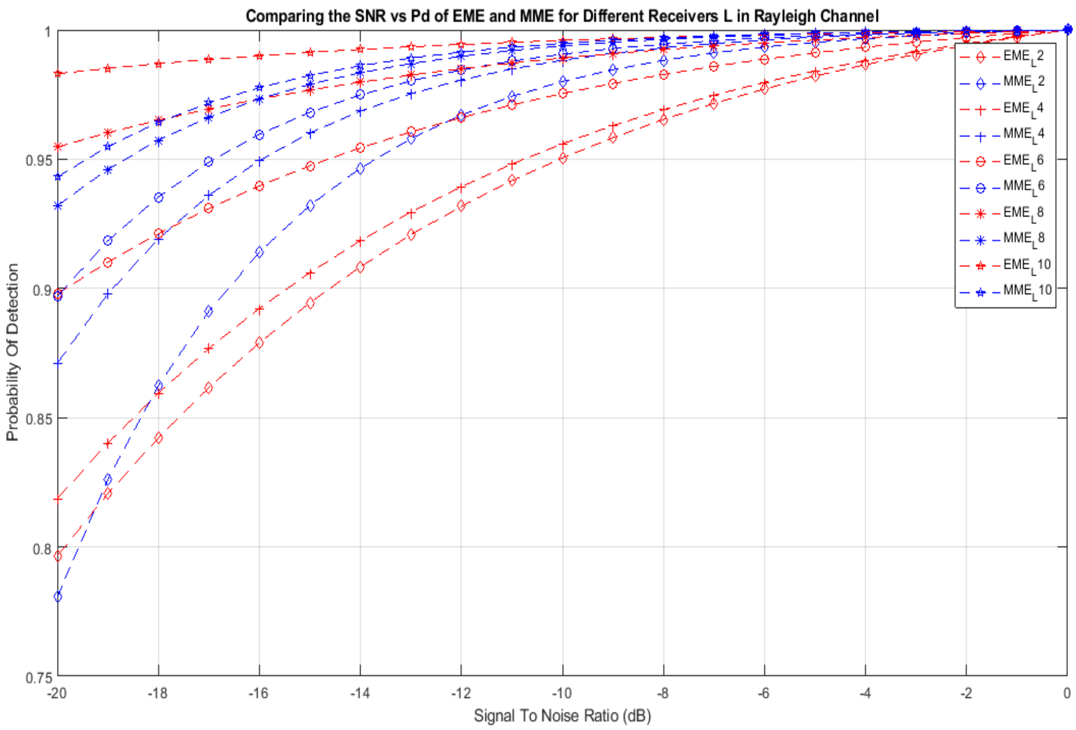

Figure 7. ROC for the Performance of MME (Blue) and EME (Red) for different receivers L.

TABLE IV. PERCENTAGE DISTRIBUTION OF P_D POINTS IN THE REGION FOR ED, MME AND EME

P_d Region	ED		MME		EME	
	P_d (Points)	P_d (%)	P_d (Points)	P_d (%)	P_d (Points)	P_d (%)
EXR	-	-	51	92.7	48	87.3
VGR	-	-	3	5.5	6	10.9
UGR	1	1.8	1	1.8	1	1.8
LGR	2	3.6	-	-	-	-
FIR	4	7.3	-	-	-	-
UPR	4	7.3	-	-	-	-
LPR	6	10.9	-	-	-	-
VPR	13	23.6	-	-	-	-
ISR	25	45.5	-	-	-	-

1) Observations and Discussions

(1) From Table 4, MME has a larger percentage of P_d points in the EXR region (92.7%) compared to the EME method which has a percentage of (87.3%). This shows that the MME detector achieves better performance than the EME. The reason for this is two folds;

(i) Both MME and EME are used to perform signal detection without the prior knowledge of the signal or noise power. The MME detection compares the decision threshold with the ratio of the maximum eigenvalue to minimum eigenvalue. The expression for the decision threshold is derived based on the asymptotic or limiting distribution of the extreme eigenvalues as developed from Random Matrix Theory (RMT). By using this exact decision threshold, the detection is better than the EME.

(ii) The EME detection compares the decision threshold with the value of the ratio of the average signal power to the minimum eigenvalue. Unlike ED, the threshold is not based on noise power. However, EME will only be highly accurate compared to MME when the average power is calculated correctly. A failure in estimating the average signal power correctly will lower its performance when compared with MME.

(2) From Fig. 7, MME achieves convergence faster than EME detection method. The MME attained convergence at -6dB while EME at -2dB. The reason for this is also the same reason given in (1).

(3) For large number ($L \geq 10$) of SU receivers, EME performed slightly better than MME at very low SNR.

With the points stated in (1), (2) and (3), this work has been able to show that the eigenvalue based methods, the MME and the EME performs significantly better than ED method in cognitive radio for the specified VHF/UHF broadcasting frequency bands in Rayleigh channel. The MME and EME were shown to be able to detect very weak primary users' signal which is the major drawback of ED method.

V. CONCLUSION

In this work, comparative analysis of two eigenvalue based (MME and EME) and energy detection (ED) sensing methods in a low SNR Rayleigh channel were performed for VHF/UHF broadcasting bands in cognitive radio network. The results show that the MME and EME detection methods performed better than the ED method under low SNR scenario in Rayleigh channels. The MME achieved convergence at a lower SNR than the EME implying that the MME has better probability of detection than the EME method under low SNR scenario.

This information will aid engineers in the design of better primary user's signal detection methods for future cognitive radio applications in a Rayleigh channel. Cognitive radio sensing network can be designed as hybrid sensing technique, combining ED with MME or EME with ED. Because of its low complexity in implementation, the ED can be used for sensing in high SNR condition while the MME or EME will be used to sense in low SNR scenario as this work has shown.

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