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Reduce Resistivity and Increase the Conductivity of Two-Dimensional Graphene with Local Electron

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Abstract- Development of two-dimensional electron gas systems started from the 8th century and now research and methods of using these systems is growing - as is the case of two-dimensional electron gas in graphene surfaces. In this article, the effect of impurity atoms on the tensor conductivity and resistivity graphene two-dimensional electron gas systems investigated. Shows electrical conductivity increases and thus the resistivity decreases.

Keywords- *Graphene; 2DEG; impurity states; Conductivity tensor; Resistivity*

I. INTRODUCTION

Development of two-dimensional electron gas (2DEG) systems started from the 8th century and now in the system appears in different positions [1]. Research and methods of using these systems is growing - as is the case of 2DEG in Graphene surfaces. [2-3]

Real systems always include the scattering centers. As in the case of two-dimensional, the weak gravitational centers leading to impure state electron and Enables to formation of a state (localized) electron affiliates [4], and its related to electron amplitude pole- Impure states - such is a function of electron energy.

The pole should be considered in the calculation of the dynamic conductivity. Important to understand the effect of these modes on the conductor. This work is explained in a localized area (Localized states, which are caused by impurities and vacancy. Significantly affect the dynamic properties of semiconductor and optical) in the high-frequency conductivity of two-dimensional electron gas appears when the spatial dispersion can be neglected.

Electron dispersion law is expected to have a contribution. Energy is equal to:

$$\varepsilon = \frac{p^2}{2m} \tag{1}$$

Where m is the electron mass and momentum p.

Scattering centers are distributed irregularly on page Z =0, and their density is n_1. (It is assumed that the impurities are distributed randomly and Point). Scattered potential at distances less effective. Potential energy of electrons in the impurity field is as follows:

$$V(\vec{r}) = V_0 \sum \delta(\vec{r} - \vec{R}_j) \tag{2}$$

V_0 is constant.

$\vec{r}$ *and* $\vec{R}$ are two-dimensional radial vectors and J is the number of Impure atoms. Concentration of impurities is assumed to be little and more the interference and impurity scattering of electrons is studied.

In the following, the effect of impurity atoms on the tensor conductivity and resistivity graphene two-dimensional electron gas systems at high frequencies $\omega \succ\succ \nu$ is investigated. [2]

II. CALCULATEING CONDUCTIVITY TENSOR AND RESISTIVITY

A. *Electrons of impurity states*

Impure surface electron in scattering centers located at the origin of the coordinate when

$$V(\vec{r}) = V_0\, \delta(\vec{r}) \;\&\; \overrightarrow{R_j} = 0 \tag{3}$$

From lifshitz equation [5-6] can be found.

$$1 - V_0\, F(\varepsilon) = 0 \tag{4}$$

Where $F(\varepsilon)$ is a real function.

$g_o(\varepsilon)$ density of impurity states excited of two-dimensional electrons. where

$$g_0(\varepsilon) = \frac{m}{2\pi\hbar^2} \;\&$$

$$F(\varepsilon) = \frac{m}{2\pi\hbar^2}\int_0^w \frac{d\varepsilon'}{\varepsilon - \varepsilon'} = -\frac{m}{2\pi\hbar^2} Ln\left|\frac{w-\varepsilon}{\varepsilon}\right| \tag{5}$$

B. Green's function with Considering the impurity states electron

The temperature of single-electron Green's function is:

$$G_{\sigma\sigma'}\left(\vec{k},\tau,\vec{k}',\tau\right)=-\left\langle T_\tau\left\{\hat{a}_{\vec{k}\sigma}(\tau)\hat{k}_{\vec{k}'a'},(\tau')\right\}\right\rangle \quad (6)$$

Where

$a_{\vec{k}\sigma}(\tau)$ is matsubar operator of the electron have been destroyed with $\vec{k}=\frac{\vec{\rho}}{\hbar}$ and Spin quantum number σ.

$\beta=\frac{1}{k_BT},\tau'\geq\beta,\tau\geq.$ - k_B boltzmann constant, T - absolute temperature, T_τ - Symbol of order time & $<.....>$ is Gibbs average symbol.

Function (6) is Related with temperature scattering Operator T [7 -8]

$$\overline{G}=\overline{G}_0+\overline{G}_0\overline{T}\overline{G} \quad (7)$$

$\overline{G}_0$ is Green's function of ineffective Impurity. With averaging (6) for positions of impurity atoms, we know Matrix (6) is diagonal for $\vec{k}$.

$$\left\langle G_{\sigma\sigma'}\left(\vec{k},\tau;\vec{k}',\tau\right)\right\rangle_c=G\left(\vec{k};\tau,\tau'\right)\sigma_{\vec{k}\vec{k}'}\delta_{\sigma\sigma'}$$

And for σ be considered diagonal. With Fourier transform for $\tau-\tau'$ [9] and from (7) equation obtains:

$$G_{\vec{k}}(\zeta_s)=G_{\vec{k}}^{(0)}(\zeta_s)+G_{\vec{k}}^{(0)}(\zeta_s)T_{\vec{k}}(\zeta_s)G_{\vec{k}}^{(0)}(\zeta_s). \quad (8)$$

Where

$$G_{\vec{k}}^{(0)}(\zeta_s)=\frac{1}{i\zeta_s-\zeta_{\vec{k}}}$$

And μ is chemical potential of electrons. And

$\zeta_{\vec{k}}=\frac{h^2k^2}{2m}-\mu$

$$\zeta_s=\frac{\pi}{\beta}(2s+1),\qquad S=0,\pm1,\pm2,...$$

When impurity atoms is little

$$T_{\vec{k}}(\zeta_s)\cong\sum_{i=1}^{N_i}t_i=N_it_k(\zeta_s)$$

Where N_i is number of impurity atoms per unit area. $(S=L^2)$

$\hat{t}-$ operator of electrons scattering in an impurity. In δ state, the potential distribution t may be found.

$$t(\zeta_s)=\frac{V_0/s}{1-\frac{V_o}{S}\sum_{\vec{k}}G_k^{(o)}(\zeta_s)} \quad (9)$$

Where

$$\sum_{\vec{k}}.....=\frac{S}{(2\pi)^2}\int d^2k$$

With Relocation in (8) & (9) equations as $i(\zeta_s)$ to $\varepsilon+iO$ Green's function delay $\oplus$ and Preliminary $(-)$, Green's function obtained as follows:

$$G_{\vec{k}}^{\pm}(\varepsilon)=G_{\vec{k}}^{(o)\pm}(\varepsilon)+\left(G_{\vec{k}}^{(o)\pm}(\varepsilon)\right)^2t^{\pm}(\varepsilon) \quad (10)$$

Where $G_{\vec{k}}=\frac{\hbar^2k^2}{2m}$ and

$$t^{\pm}(\varepsilon)=\frac{V_0/s}{1-\frac{V_o}{S}\sum_{\vec{k}}G_k^{(o)\pm}(\varepsilon)} \quad (11)$$

$$G_k^{(o)\pm}(\varepsilon)=\frac{1}{\varepsilon-\varepsilon_k\pm iO} \quad (12)$$

By using from sakhusk formula

$$\frac{1}{x+iO}=\rho\frac{1}{x}\mp i\pi\delta(x) \quad (13)$$

If we take ρ (mark important parameter)

$$\frac{1}{S}\sum_{\vec{k}}G_k^{(o)\pm}(\varepsilon)=F(\varepsilon)\mp i\pi g_o(x) \quad (14)$$

As a result

$$t^{\pm}(\varepsilon)=\frac{V_0/s}{1-V_o\left[F(\varepsilon)\mp i\pi g_o(x)\right]} \quad (15)$$

Green average function (8) & (10), we have the following Spectral form.

$$G_k^{(o)\pm}(\varepsilon)=\int_{-\infty}^{+\infty}d\varepsilon'\frac{\rho_k(\varepsilon')}{\varepsilon-\varepsilon'\pm iO} \quad (16)$$

Where $\rho_{\vec{k}}(\varepsilon')$ spectral density of the Green's functioned and is equal to:

$$\rho_{\vec{k}}(\varepsilon')=\mp\frac{1}{\pi}\mathrm{Im}\,G_{\vec{k}}^{\pm}(\varepsilon) \quad (17)$$

With Placing (10) in (17), we have

$$\rho=\rho_o+\delta\rho \quad (18)$$

Where

$$\rho_{\vec{k}}^{(o)}(\varepsilon)=-\frac{1}{\pi}\mathrm{Im}\frac{1}{\varepsilon-\varepsilon_k+iO}=\delta(\varepsilon-\varepsilon_k)$$

And obtains

$$\delta\rho_{\vec{k}}(\varepsilon)=\frac{1}{\pi}\frac{1}{\left(\varepsilon-\varepsilon_{\vec{k}}\right)^2}|V_o|n_i\delta\left[1-V_oF(\varepsilon)\right] \quad (19)$$

From (19) we Obtains [4]

$$\delta\rho_{\vec{k}}(\varepsilon)=n_i\sum_i\frac{r_\ell}{\left(\varepsilon-\varepsilon_{\vec{k}}\right)^2}\delta(\varepsilon-\varepsilon_\ell) \quad (20)$$

Where

$$r_\ell = \frac{1}{|F'(\varepsilon_\ell)|}, F' = \frac{dF}{d\varepsilon} \tag{21}$$

Calculation of scattering amplitude impurity electrons is taken by combining over Localized levels. With Placing (5) in (21), we have

$$r = \frac{2\pi\hbar^2}{m}|\varepsilon_\ell|\left(1-\frac{h\omega_g}{\omega}\right) \tag{22}$$

Where $|\varepsilon_\ell| = \hbar\omega_g$.

For $\omega_o << \omega$

$$r = \frac{2\pi\hbar^2}{m}|\varepsilon_\ell| \tag{23}$$

C. Conductivity tensor in High Frequency and Resistivity

To calculate the conductivity tensor of two dimensional electron gas systems (15) affected by impurities, we use from Kubo Formula [9]

$$\sigma_{\alpha\beta}(\omega) = i\frac{e^2 n}{m\omega}\delta_{\alpha\beta} + \frac{1}{\omega s}p^+_{\alpha\beta}(\omega) \tag{24}$$

Where p^+ is two-particle Green's function of electrons.

$$p^+_{\alpha\beta}(\omega) = -\frac{i}{\hbar}\int_0^\infty dt\exp(i\omega t) << \left[\bar{J}_\alpha(t), \hat{J}_\beta(o)\right] >>_c$$
$$\hat{J}(t) = 2\frac{e\hbar}{m}\sum \vec{k} a^+_{\vec{k}}(t)\,\hat{a}_{\vec{k}}(t) \tag{25}$$

And $\hat{J}(t)$ operator of Heisenberg current density, e Electron charge, n Electron density, $\left[\hat{a},\hat{b}\right] = \hat{a}\hat{b} - \hat{b}\hat{a}$. Spatial distribution of conductivity is not calculated here. By putting (25) in (24), Relation of tensor $\sigma_{\alpha\beta}$ with two-particle Green's function of electrons Obtained. The last calculate with the help of Vike theory [9] is performed. Then result is taken by assuming that the electrons do not affect each other, we can to use from product of two single-particle Green's function. From approximation [9] namely instead of using the product of the averages, we use from the average of the products:

$$<G_1 G_2> \approx <G_1>_c <G_2>_c \tag{26}$$

Therefore with matsubar technique [9]

$$\rho_{\alpha\beta}(\omega_n) = 2\left[\frac{e\hbar}{m}\right]^2 \sum_k k_\alpha k_\beta . k_B T \sum_{s=-\infty}^{\infty} G_{\vec{k}}(\zeta_s - \hbar\omega_n) \tag{27}$$

Where $G_{\vec{k}}(\zeta_s)$ averaged Green's function (8) and $\omega_n = \frac{2\pi m}{\hbar\beta}$ matsubar frequency (here n Electrons density).we put (27) in (16). We apply the sum overs. And the decomposition (27) with a discrete set of points on the real axis frequency does. Obtained:

$$\sigma_{\alpha\beta}(\omega) = i\frac{e^2 n}{m\omega}\delta_{\alpha\beta} + i\frac{e^2}{\omega s}.2\frac{\hbar^2}{m^2}$$
$$.\sum_{\vec{k}} k_\alpha k_\beta \int_{-\infty}^{\infty} d\varepsilon \int_{-\infty}^{\infty} d\varepsilon' . \rho_{\vec{k}}(\varepsilon)\rho_{\vec{k}}(\varepsilon') . \frac{f(\varepsilon') - f(\varepsilon)}{\varepsilon' - \varepsilon - \hbar\omega - iO} \tag{28}$$

$f(\varepsilon)$ is the Fermi function. By putting (18) in (28) and use from

$$\rho_{\vec{k}}(\varepsilon)\rho_{\vec{k}}(\varepsilon') \approx \rho^{(0)}_{\vec{k}}(\varepsilon)\rho^{(0)}_{\vec{k}}(\varepsilon') +$$
$$\delta\rho_{\vec{k}}(\varepsilon)\delta(\varepsilon' - \varepsilon_{\vec{k}}) + \delta\rho_{\vec{k}}(\varepsilon')\delta(\varepsilon - \varepsilon_{\vec{k}})$$

Where

$$\delta\rho_{\vec{k}}(\varepsilon)\delta(\varepsilon' - \varepsilon_{\vec{k}}) + \delta\rho_{\vec{k}}(\varepsilon')\delta(\varepsilon - \varepsilon_{\vec{k}}) \approx n_i$$

With the conversion of conductance to linear format according to n_i, the high frequency conductivity with regard to independent impurities, approximately is equal to

$$\sigma(\omega) = \sigma_0(\omega) + \delta\sigma\ (\omega) \tag{29}$$

That $\sigma_0(\omega)$ is a certain proportion [7], and obtained in the absence of impurities and $\sigma(\omega)$ is contribution of impurity levels. In the absence of the rider, the spatial distribution of localized levels cause that

$$\sigma_{\alpha\beta}(\omega) = \frac{1}{2}\sigma\ (\omega)\delta_{\alpha\beta}$$

From (28) obtains

$$\delta\sigma_{\alpha\beta} = i\frac{e^2}{\omega s}2\frac{\hbar^2}{m^2}\times$$
$$\sum_{\vec{k}} k_\alpha k_\beta \int_{-\infty}^{\infty} d\varepsilon \left\{\frac{f(\varepsilon_{\vec{k}}) - f(\varepsilon)}{\varepsilon_{\vec{k}} - \varepsilon - \hbar\omega - io} + \frac{f(\varepsilon) - f(\varepsilon_{\vec{k}})}{\varepsilon - \varepsilon_{\vec{k}} - \hbar\omega - io}\right\} \times \delta\rho_{\vec{k}}(\varepsilon) \tag{30}$$

With calculating (20) from (30), we obtain contribution of localized levels in High Frequency.

$$\delta\sigma\ (\omega) = i\frac{e^2}{\omega s}\frac{\hbar^2}{m^2}\times\sum_l l\sum_{\vec{k}}\frac{k^2}{(\varepsilon_{\vec{k}} - \varepsilon_1)^2}\left[f(\varepsilon_l) - f(\varepsilon_{\vec{k}})\right]\times$$
$$\left[\frac{1}{\varepsilon_l - \varepsilon_{\vec{k}} - \hbar\omega - io} + \frac{1}{\varepsilon_l - \varepsilon_{\vec{k}} + \hbar\omega - io}\right] \tag{31}$$

In relocation, means instead of gather series of k and Integration in terms of energy, we obtain from (31):

$$\delta\sigma\ (\omega) = i\frac{e^2}{\omega s}\frac{\hbar^2}{m^2}\times\sum_l l\int_0^\infty d\ \varepsilon\sum_{\vec{k}}\frac{\varepsilon}{(\varepsilon - \varepsilon_1)^2}\left[f\ (\varepsilon_l) - f\ (\varepsilon)\right]\times$$
$$\left[\frac{1}{\varepsilon_l - \varepsilon - \hbar\omega - io} + \frac{1}{\varepsilon_l - \varepsilon_{\vec{k}} + \hbar\omega - io}\right] \tag{32}$$

By using from (13) to separate the real and virtual (32), We have

$$\mathrm{Re}\ \delta\sigma\ (\omega)=\frac{e^{2}n_{i}}{\hbar^{4}\omega^{3}}\times \sum_{l} l\theta(\varepsilon_{l}-\hbar\omega)(\varepsilon_{l}-\hbar\omega)\left[f(\varepsilon_{l}-\hbar\omega)-f(\varepsilon_{l})\right]+(\omega\to-\omega) \tag{33}$$

$$\mathrm{Im}\ \delta\sigma\ (\omega)=\frac{e^{2}n_{i}}{\pi\ \hbar\omega^{3}}\times \sum_{l} l\int_{0}^{\infty}d\varepsilon\frac{\varepsilon}{(\varepsilon-\varepsilon_{l})^{2}}\left[f(\varepsilon_{l})-f(\varepsilon)\right]\left[\frac{1}{\varepsilon_{l}-\varepsilon-\hbar\omega}\right]+(\omega\to-\omega) \tag{34}$$

$(\omega\to-\omega)$ Means that we have sentence same first part by changing the sign of the angular frequency. θ - Heaviside step function.

From (33) and (34), we can to see that $\mathrm{Re}\,\delta\sigma(\omega)$ is the function of frequency even and $\mathrm{Im}\,\delta\sigma(\omega)$ is the function of frequency odd.

Integral (34)for is to be calculated. As a result, Contribution of a localized level for high-frequency conductivity of electrons is:

For $T\succ 0$

$$\mathrm{Re}\ \delta\sigma\ (\omega)=\frac{e^{2}n_{i}}{\hbar^{4}\omega^{3}}r\times \left[f(\varepsilon_{l})-f(\varepsilon_{l}+\hbar\omega)\right](\varepsilon_{l}+\hbar\omega)\ \theta\ (\varepsilon_{l}+\hbar\omega)+(\omega\to-\omega) \tag{35}$$

And for $\varepsilon_{1}\prec\varepsilon_{F}$ & $\varepsilon_{1}\succ\varepsilon_{F}$ & $T=0$ the answer is imaginary.

Here ε_{F} is Position of the Fermi energy in the field is affected by the impurities and $\varepsilon_{l}=-\left|\varepsilon_{l}\right|$.

These Formulas use for $\omega\succ\succ\nu$ and ν is the electron collision frequency.

The Curve (1) shows the function $\psi(x)$ which as follows

$$\psi(x)=\frac{\pi\hbar^{3}\omega_{g}^{2}}{e^{2}n_{i}r}\mathrm{Re}\,\delta\sigma(\omega) \tag{36}$$

$x=\frac{\omega}{\omega_{g}}X$ is the dimensionless frequency. In figure dependence of $\psi(x)$ in unit of Static conductivity $\frac{e^{2}n}{m\nu}$ than X is observed.

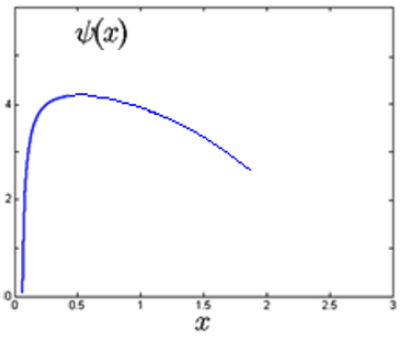

Figure 1. The function $\psi(x)$ than X

In Figure can be seen that the real part $\psi(x)$ has a maximum. Localized electrons are active in the conductivity area. Can be obtained the following relation:

$$\frac{\left[\mathrm{Re}\,\delta\sigma(\omega)\right]_{\max}}{\sigma_{0}}=\psi_{\max}\frac{e^{2}n_{i}r}{\pi\hbar^{3}\omega_{g}^{2}\sigma_{0}} \tag{37}$$

Where ($\sigma_{0}=\frac{ne^{2}\tau}{m}$) the standard contribution in two-dimensional electron gas [7] and τ Is the electron mean free medium.

With Placing (22) in (37) and σ_{0} value, we have

$$\frac{\left[\mathrm{Re}\,\delta\sigma(\omega)\right]_{\max}}{\sigma_{0}}=\psi_{\max}2\frac{n_{i}/n_{e}}{\omega_{0}\tau} \tag{38}$$

If $\psi_{\max}\approx 0.41$ (Obtained from Fig.1) ، $\frac{n_{i}}{n_{e}}=0.5$ ، $\omega_{g}\tau=10$ ،for (38) (typical values for graphene : $\frac{\varepsilon_{F}}{\hbar\omega_{g}}=0.5$ ، $\frac{T}{\hbar\omega_{g}}=0.1$ ، $m=10^{-28}g$ ، $\left|\varepsilon_{l}\right|=0.01eV$), We will have

$$\frac{\left[\mathrm{Re}\,\delta\sigma(\omega)\right]_{\max}}{\sigma_{0}}=0.041=4.1\% \tag{39}$$

And since the resistivity is $\rho\propto\frac{1}{\sigma}$, If ρ_{0} related to the case without impurities and ρ_{1} for the case where the conductivity increases, thus we have:

$$\frac{\rho_{1}}{\rho_{0}}\propto\frac{\sigma_{0}}{\sigma_{1}}\propto\frac{\sigma_{0}}{\sigma_{0}+0.041\sigma_{0}}=0.96 \tag{40}$$

As a result, resistivity is reduced relative to the initial state.

III. Conclusions

In this paper, the effect of impurity atoms on a two-dimensional electron gas system in the high-frequency conductivity tensor of graphene was investigated. States of electrons in the field of Impurity atoms is considered.

Threshold behavior of the real part of the tensor near the threshold frequency is shown. Numerical calculations for the graphene that includes a two-dimensional electron gas in the vicinity of the silicon oxide surface have been performed. And shows electrical conductivity increases as much as 4.1 percent and and thus the resistivity decreases.

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