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Quantized Compressed Sensing for FPCG Signals

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Abstract- As the fPCG signals are widely used to monitor the condition of the fetus health, it needs to be save or transmit with lower costs; thus the compression of fPCG with Compress Sensing (CS) method is a beneficial method to reach this purpose. As the fPCG signal is not spares in time domain, it should be brought to another orthogonal space. Because of the structured sparsity in this new space, the used CS method should be adapted to the fPCG signal's conditions which lead to the Shuffled CS (S-CS) method. In this article we innovate and develop the Quantized S-CS (QS-CS) method to improve the compression rate of the S-CS. As the reconstruction process is a convex optimization problem, it extremely limits the quantization noise in QS-CS. The simulation results show that the proposed QS-CS method, along with providing a greater CR, has equivalent reconstruction performance to the primer S-CS method.

Keywords- *Compressed Sensing, fPCG signal, Quantization, orthogonal spaces, reconstruction methods*

I. INTRODUCTION

Since medical care during the pregnancy time plays an impressive role to prevent the fetus anomalies, the mother and fetus health should be monitored clinically over three separate periods each which lingers three months. Measuring the fetus heart beat rate starts in the twenty fourth week of pregnancy and is determined daily. One of the safest methods for this purpose is recording the heart beat sound which contributes the Fetal Phonocardiography (fPCG) methods [1]-[4]. Sometimes the doctors need to have this fPCG signals daily, thus the mother should be able to tape it and send it to the clinic as easily as possible. Therefore, the recorded data may contain massive information that its storage and transmission is very expensive. To decrease the expenditure of storage and transmission of fPCG signal, it is inevitable to develop a method to compress the recorded data. In [5]-[10] the authors proposed efficient methods for compression of the clinical signals including Electroencephalography (EEG) and Electrocardiography (ECG) signals. Also an innovative fECG denoising and diagnosis methods is developed in [11]. To our knowledge, no work has been done for fPCG signal compression and it is an open problem in biomedical engineering.

The Compressed Sensing (CS) method firstly presented in [12] and [13] and is claimed that can compress the signals with a rate lower than the Nyquist rate and reconstruct the original signal efficiently. Also, the authors in [14] and [15] employed a quantization to maintain furthers compression rate of CS.

On the other hand, since especial conditions of fPCG signals, the CS method should be adapted to fPCG conditions for signal compression; in [16] we developed a Shuffled CS (S-CS) method for fPCG signal compression. In this article we invent the novel Quantized S-CS (QS-CS) method to improve the compression rate of the previous S-CS for fPCG signals; in addition, we investigate the quantization effects on the reconstruction processes.

The remaining of this article is organized as follows. Section II includes the general CS model; the implementation of CS fPCG compression is presented in Section III in which the S-CS and the proposed QS-CS models are developed step by step. The simulation results are revealed in Section IV. Finally Section V holds the conclusions.

II. CS MODEL

Generally CS is a lossless compression method but sometimes to reach the compression goals it may be a lossy one. The CS is involved with two main stages; firstly, the signal compression stage and secondly, the signal reconstruction from the compressed one in the first stage. Both parts are elaborated below.

A. Signal Compression

Let's suppose $x(t)$ is the fPCG signal which is going to be compressed by CS method. If we define $X \triangleq (x_i)_{i=1}^{n} \in R^n$ as the column vector contained the Nyquist samples of $x(t)$, there is a potential to compressed this vector with CS method if the sparsity of X is confirmed. The measure to determine the level of sparsity of a vector is sparsity factor λ, which is defined as [17]

$$\lambda = \frac{\sqrt{n} - \frac{\|X\|_1}{\|X\|_2}}{\sqrt{n} - 1}, \tag{1}$$

where $\|X\|_1$ and $\|X\|_2$ are the ℓ_1 and ℓ_2 norms of vector X respectively. It is noticeable that $0 \le \lambda \le 1$ in which $\lambda = 0$ illustrates that X is quite non-sparse and $\lambda = 1$ depicts that X is an absolute spares vector. To prove that X is as sparse as is needed, λ should be restricted as $k \le \lambda \le 1$, in which k is the specialized minimum acceptable sparsity factor.

In some cases X does not satisfy the sparsity restrictions, thus it should be brought to another orthogonal space in which its coefficients are calculated as

$$s_j = \sum_{i=1}^{n} \psi_{j,i} x_i. \quad (2)$$

where s_j is the extension of X in the j th dimension of the orthogonal space. If $\psi_{j,i}$ for $i = 1, \ldots n$ and s_j contribute the j th row of the $\mathbf{\Psi}$ (the orthogonal space transformation matrix) and j th element of S respectively, it would be

$$S = \mathbf{\Psi} X . \quad (3)$$

The orthogonal space should be selected so that S is as sparse as is needed, meaning that $k \le \lambda_S \le 1$, where

$$\lambda_S = \frac{\sqrt{n} - \frac{\|S\|_1}{\|S\|_2}}{\sqrt{n} - 1}. \quad (4)$$

After being certain that S is sparse enough, it is appropriate to be compressed by CS process. The compressor matrix $\mathbf{\Phi}_{m \times n}$ is known as the sensing matrix in CS. The number of its rows is lower than number of its columns ($m < n$) and is considered to compress S according to

$$Y = \mathbf{\Phi} S, \quad (5)$$

where vector Y with size $m \times 1$ is the compressed version of S that can be efficiently stored or transmitted with lower cost rather than X because $m < n$. In CS, the best choice for the sensing matrix is the $\mathbf{\Phi}$ whose each elements are set with random values with white zero mean Gaussian distribution [18]. From (5) it is $Y = \mathbf{\Phi \Psi} X$, thus by defining $\mathbf{C} = \mathbf{\Phi \Psi}$ as the CS matrix, we have

$$Y = \mathbf{C} X. \quad (6)$$

As the compressed vector Y is supposed to be saved or transmit, the Compression Rate (CR) of the CS matrix, also is defined as

$$CR = \frac{n}{m}. \quad (7)$$

which is the ratio of $\mathbf{C}$'s number of columns to its number of rows (the uncompressed vector size to compressed vector size ratio).

B. *Signal Reconstruction*

After compression of X into Y , for monitoring the patient heart beat rate in our system, it is necessary to reconstruct the original vector X from Y . But rehabilitating n elements of X from m elements of Y is one of the crucial aspects of CS because $m < n$. The bright side of CS reconstruction is the sparsity of S which guarantees the uniqueness of $\hat{S}$, the reconstructed version of S . In the reconstruction step, it is assumed that $\hat{S}$ is the sparsest vector which satisfies $Y = \mathbf{\Phi} \hat{S}$. According to [19], the efficient way to describe the reconstruction problem is the so-called Basis Pursuit (BP) problem expressed as

$$\hat{S} = \arg\{\min_S \|S\|_1\}, \text{ subject to } \; Y = \mathbf{\Phi} S. \quad (8)$$

As is obvious that the BP is an optimization problem which is based on the minimization of the ℓ_1 norm of S . In other words, according to BP problem, the sparsity attribute of desired vector is considered as minimizing its ℓ_1 norm. The S with the minimum ℓ_1 norm would be the answer of the reconstruction stage.

So far, there are many numerical algorithms to solve the BP problem. The challenges of these algorithms are the speed of convergence and the calculation complexity [20]. One of the well-known fast algorithms is Projected Proximal Point Algorithm (ProPPA) [21] which is considered as the reconstruction algorithm in this article. After having $\hat{S}$ from (8), the reconstructed version of fPCG signal $\hat{X}$ would be available as

$$\hat{X} = \mathbf{\Psi}^{-1} \hat{S}. \quad (9)$$

To investigate the performance of reconstruction, the Percentage Root mean-square Difference (PRD) is a measure for determining the reconstruction errors [22]

$$\% PRD = 100 \sqrt{\frac{(X - \hat{X})^T (X - \hat{X})}{X^T X}}. \quad (10)$$

To evaluate the performance of the implemented CS, the compression rate CR from (7) and the reconstruction error PRD are considered. The higher CR along with lower PRD shows the better CS performance; and vice versa, the lower CR and higher PRD reveals the worst performance. Of course there would be a tradeoff between these two measures which should be specialized for particular purposes.

III. CS Method for fPCG Signal Compression

Because of especial features of fPCG signals, the CS method should be designed considering these features, in order to yields the best performance. Thus in [16] we first developed our Shuffled CS (S-CS) method and in this article we invent the Quantized S-CS (QS-CS) method to improve the performance of S-CS for compression of fPCG signals. Therefore, in this section we first present the S-CS and then develop the QS-CS adapted to compress fPCG signals.

A. *The S-CS Method*

Consider a typical fPCG signal with length $n = 1024$ which is sampled version of continues recorded signal with rate 1 kHz, illustrated in Fig. 1. The sparsity factor of this signal is low ($\lambda = 0.39$). As is obvious, fPCG signal is not sparse enough at time domain; thus, first of all, we are obligated to utilize the most appropriate orthogonal space in which the fPCG coefficients are as sparse as possible.

According to [16] the Periodized Orthogonal (PO), Fast Analysis Operator for Periodized Orthogonal Wavelets Dictionary Normalized by Total Variation (POTV) and Discrete Cosine Transform (DCT) spaces provide the sparsest coefficients for the actual fPCG signal. Then, in this article we consider PO, POTV and DCT to build the $\mathbf{\Psi}$ (the space orthogonal matrix).

Fig. 2 depicts the PO, POTV and DCT coefficients of the typical fPCG signal of Fig.1. The sparsity factors for PO, POTV and DCT coefficients are rather high ($\lambda = 0.58$, 0.47 and 0.66 respectively). It is clear because of larger λ the resulted coefficients in orthogonal spaces are sparser than the original signal in time domain.

On the other hand, it is disclosed that the fPCG coefficients in the mentioned spaces are structured sparse. In other words, all the coefficients aggregate into the one side of the vector S. From the reconstructor algorithms point of view, the structured-sparse coefficient is not acceptable [21]; therefore, to make the coefficient vector as an appropriate case for the reconstructor algorithms and having the best performance, we use a shuffler matrix $\mathbf{\Theta}$ which diffuses the coefficients through all the vector positions. Thus $\mathbf{\Theta}$ is so that in which each row or column has just one non-zero element. This non-zero element is set to 1 which is randomly posed in the matrix $\mathbf{\Theta}$. Eq (11) shows an example of shuffler matrix when $n = 4$

$$\mathbf{\Theta} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}. \quad (11)$$

The resulted coefficient vector after shuffling yields to

$$S' = \mathbf{\Theta\Psi} X. \quad (12)$$

By defining $\mathbf{\Psi}' = \mathbf{\Theta\Psi}$ as the new orthogonal space, we have

$$S' = \mathbf{\Psi}' X. \quad (13)$$

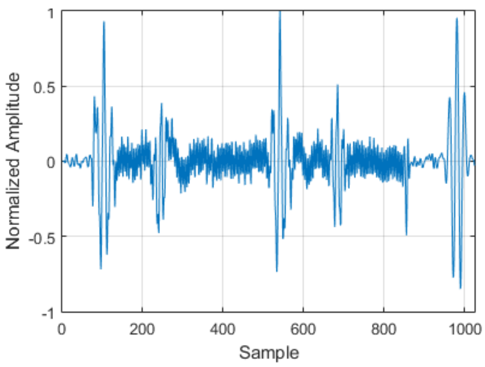

Figure 1. Typical fPCG signal with length $n = 1024$, sample rate 1 kHz and the sparsity factor $\lambda = 0.39$.

Fig. 3 shows the diffused vector S' for three mentioned spaces. It is clear that the resulted vector can be considered as a non-structured sparse vector which is the most prominent needed attribute for numerical BP problem solvers like ProPPA [21]. Now, according to (5), we have $Y' = \mathbf{\Phi} S'$ which leads to

$$Y' = \mathbf{C}' X \quad (14)$$

where $\mathbf{C}' \triangleq \mathbf{\Phi\Psi}'$ is the new CS matrix with shuffler.

For reconstruction of the compressed vector, according to (8), $\hat{S}'$ would be

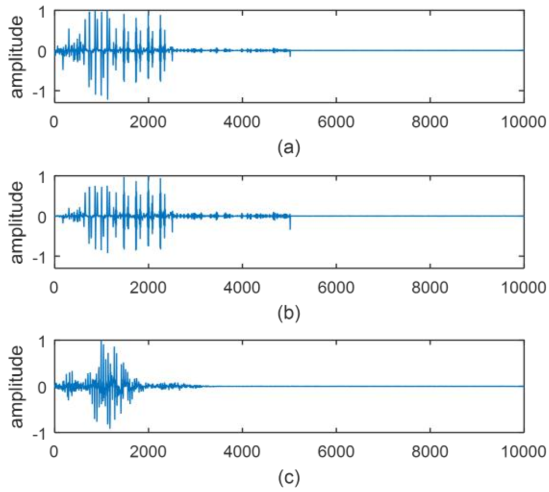

Figure 2. The PO (a), POTV (b) and DCT (c) coefficients of the typical fPCG signal in Fig. 1 with the sparsity factor $\lambda = 0.58$, 0.47 and 0.66 respectively.

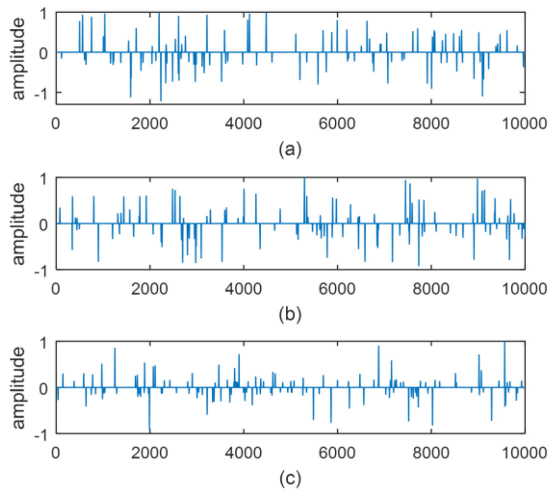

Figure 3. Diffused vector of fPCG signal coefficients in PO (a), POTV (b) and DCT (c) spaces.

$$\hat{S}' = \arg\{\min_{S} \|S\|_1\}, \text{ subject to } Y' = \mathbf{\Phi} S. \quad (15)$$

Here because of the acknowledged sparsity of S' by shuffling, it is consented that $\hat{S}'$ would be the best possible reconstructed vector of S'. After finding $\hat{S}'$, $\hat{X}$ can be easily rehabilitated as

$$\hat{X} = \mathbf{\Psi}'^{-1}\hat{S}'. \tag{16}$$

B. *The Proposed QS-CS Method*

In the S-CS method in the previous section, as mentioned in (7), the compression rate, *CR* is defined as the ratio of size X to the size of Y' (n/m). Here in the QS-CS method, to improve the *CR* of the S-CS, we implement a quantization over the Y' coefficients to limit the levels taken by the amplitude of the resulted vector. The quantized vector is defined as Y_{qu}

$$Y_{qu} = f_L(Y'), \tag{17}$$

where f_L is the quantizer function which enforces the input vector coefficients amplitudes into the maximum L levels. Fig. 4 illustrates an example of Y' and Y_{qu} with $m = 32$ and $L = 8$. Subsequently the limited levels of Y_{qu} coefficients, of course, need very fewer bits to be saved or transmit rather than Y'. Thus, the new definition of compression rate would be

$$CR = \frac{n}{m} \times \frac{b_\infty}{b_L}, \tag{18}$$

in which b_∞ and b_L are the least bit number needed for presenting each element of Y' and each element of Y_{qu} respectively. Suppose y' is an arbitrary element of Y'. If we present y' as

$$y' = \pm\Upsilon \times 10^{-r} \tag{19}$$

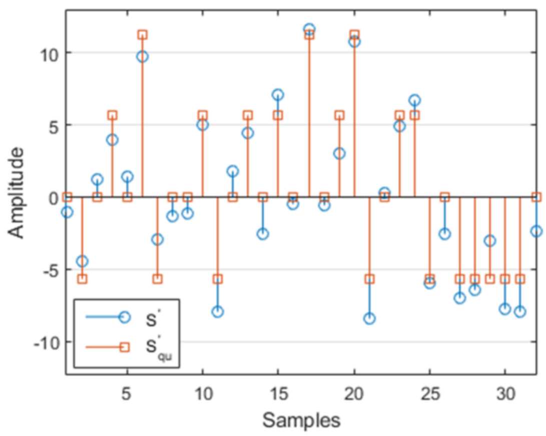

Figure 4. An example of original and quantized coefficients for $m = 32$ and $L = 8$.

in which Υ and r are integer numbers. Therefore, according to the computation scales

$$b_\infty = \mathrm{ceil}(\log_2(\max(y') \times 10^r)) + 1, \tag{20}$$

where the operator ceil(.) rounds the inputs to the nearest integer greater than or equal to the input. The term $\max(y')$ means the maximum possible value of y'. Also the term +1 presents the bit containing the sign of y'. After quantization, the needed bit for presenting each element of quantize vector Y_{qu} would be

$$b_L = \log_2(L). \tag{21}$$

On the other hand, after quantization, the BP problem for reconstruction is stated as

$$\hat{S}'_{qu} = \arg\{\min_S \|S\|_1\}, \text{ subject to } Y_{qu} = \mathbf{\Phi}S. \tag{22}$$

Also to find the time domain signal according to (16) we have

$$\hat{X}_{qu} = \mathbf{\Psi}'^{-1}\hat{S}'_{qu}. \tag{23}$$

In spite of the quantization benefits over the value of *CR* (because $b_L < b_\infty$), it imposes a noise into the reconstruction process and leads to the increase of the *PRD*. For instance, if L is a small number, the *CR* rises significantly; in contrast, because of the distortion of Y_{qu} in comparison with Y', the *PRD* of the reconstructed signal would be raised too which may leads to refuse the results of the proposed method.

To investigate the effect of quantization noise into the reconstructed signal, let's define the quantization noise as

$$E_Y = Y' - Y_{qu}. \tag{24}$$

Also, we suppose $E_Y = \mathbf{\Phi}E_S$ where E_S is the quantization noise effect contaminating the uncompressed coefficients. On the other hand, the BP problem of (22) by employing the transformation $S \to S + E_S$, is rewritten as

$$\hat{S}'_{qu} = \arg\{\min_{S+E} \|S + E_S\|_1\}, \quad \text{subject to } Y_{qu} = \mathbf{\Phi}(S + E_S). \tag{25}$$

Since for each two arbitrary vectors S and E_S, we always have $\|S + E_S\|_1 \le \|S\|_1 + \|E_S\|_1$, the problem of (25) is broken into two simpler optimization problems

$$\begin{aligned}\hat{S}'_{qu} &= \arg\{\min_{S+E_S} \|S + E_S\|_1\}, \text{ subject to } Y_{qu} = \mathbf{\Phi}(S + E_S)\\ &= \arg\{\min_{S,E_S}(\|S\|_1 + \|E_S\|_1)\}, \text{ subject to } Y' - E_Y = \mathbf{\Phi}(S + E_S)\\ &= \arg\{\min_S \|S\|_1\}, \text{ subject to } Y' = \mathbf{\Phi}S\\ &+ \arg\{\min_{E_S} \|E_S\|_1\}, \text{ subject to } E_Y = \mathbf{\Phi}E_S.\end{aligned} \tag{26}$$

The first term of the resulted problem is similar to BP problem of (15) (which leads to the sparsest signal) and the second term, $\arg\{\min_{E_S}\|E_S\|_1\}$, subject to $E_Y = \mathbf{\Phi} E_S$, reveals that the sparsest additive noise vector E_S contaminates the reconstructed vector $\hat{S}'_{\mathrm{qu}}$. Based on this argument, in the solution of BP problem of (22), the contaminating quantization noise, E_Y would have the minimum possible effect on the resulted $\hat{S}'_{\mathrm{qu}}$. Therefore, in the proposed QS-CS method the quantization process of (17) influences on the reconstruction PRD would be insignificant for appropriate value of L. In next section, the simulation results, also, confirm this argument.

IV. SIMULATION RESULTS

In this part we implement the S-CS and proposed QS-CS methods, to compress and reconstruct the fPCG signals and survey the performance of these methods over PO, POTV and DCT spaces. It is mentionable that the database which the fPCG signals are adopted is [23] which contains variety of fPCG signals with different Signal to Noise Ratios (SNR). In these simulations the fPCG signals with duration 60 minutes and SNR= -10, -8 and -4 dB are considered and the presented results of this section are the average outcomes for these three SNRs. The shuffler matrix is organized as discussed before with non-zero elements distributed with uniform probability distribution function. Also the 256×1024 sensing matrix is built with white zero mean Gaussian elements. It is noticeable that after making shuffler and sensing matrix, both of them are fixed in the system with no changes during compressing and reconstructing processes. In this section, the reconstruction algorithm to solve the BP problem is ProPPA which has a very fast and low complexity convergence [21]. When using this algorithm, more sparsity of signals results lower reconstruction errors.

For the S-CS, according to the sensing matrix size ($m \times n$), $CR = 4$. In addition, in our simulations empirically we used $b_\infty = 9$ bits (because we consider $\max(y') \approx 16$ and $r = 1$) to present each element of Y' and chose the number of quantization levels as $L = 8$; thus $b_L = 3$ bits. Consequently, for the QS-CS method $CR = 12$. As is obvious, in QS-CS the CR is significantly greater than that of S-CS. Also to investigate the performance of the S-CS and QS-CS methods, we pass the normalized vector S' coefficients form a threshold [16]. Since the coefficients lower that threshold are set to zero, larger threshold leads to sparser resulted S'. On the other hand, great threshold may eliminates the significant coefficients of S' and causes to the wrecked reconstructed signal. Thus, the value of threshold adjusts a crucial tradeoff between sparsity and losing the significant coefficients during CS process which both of them affect the PRD.

Fig. 5, Fig. 6 and Fig. 7 compare the reconstruction PRD for S-CS and QS-CS versus the value of threshold (T), when using PO, POTV and DCT spaces respectively. Firstly, as is obvious, in each three of these figures the PRD's of S-CS and QS-CS are approximately equal, which proves that the quantization noise has the least possible effect on the reconstruction process. Secondly, it is clear that for each space when $T = 0$, the S-CS and QS-CS methods have rather high PRD. By increasing T, the PRD decreases to a certain point with the minimum PRD for each space. This threshold with the minimum PRD is considered as the optimal threshold (T_{opt}). As mentioned before, the reason of error degradation is the more sparsity coming from the elimination of unnecessary coefficients by passing through the threshold. This phenomenon improves the performance of ProPPA. After the T_{opt} point, by increasing T the PRD start to increase again. This occurrence comes from the fact that large T's eliminates the significant coefficients of S' from (13) and leads to the threshold passed coefficients do not resemble the original coefficients and causes the damages on final reconstruction. Therefore, T_{opt} is the best choice for the threshold in the S-CS and QS-CS methods. Meanwhile, Table. 1 illustrates the CR, T_{opt} and the related PRD for S-CS and QS-CS methods in three spaces.

Also, note that the amount of needful bits to present the uncompressed fPCG signal with 60 minutes duration and 1 kHz sample rate, is 30.9 Mbit, for the compressed signals by S-CS and QS-CS it is 7.72 Mbit and 2.57 Mbit respectively. It is clear that the QS-CS with a little PRD, is able to diminish the burden of necessary bits to save or transmit the fPCG signal significantly.

V. CONCLUSIONS

In this article we developed the QS-CS method for compression of fPCG signals. This method is based on previous S-CS method in which sparsity of the compressing signal is the vital attribute. To provide this sparsity, the time domain signal is brought into the especial orthogonal spaces in which the signal coefficients are sparse enough. For more compression rate, the uncompressed coefficients are passed form an optimal threshold that makes them sparser. This more sparsity guarantees the accurate reconstruction of compressed signal when using greater compression rate. In QS-CS, to reach the extremely more compression rate, we also imposed a quantization on the compressed signal to adjust them into limited allowed amplitudes. We showed that because of the nature of reconstruction optimization problem, the quantization noise has the least impact on the final reconstructed signal. Therefore, on one hand, the QS-CS provides significant more compression rate rather than S-CS; on the other hand, its reconstruction error is approximately equal to that of the S-CS.

TABLE I. THE VALUE OF CR, T_{opt} AND MINIMUM PRD OF PO, POTV AND DCT SPACES ACCORDING TO S-CS AND QS-CS METHODS FOR FPCG SIGNAL.

	CR		**T_{opt}**		***PRD***	
CS method / Space	S-CS	QS-CS	S-CS	QS-CS	S-CS	QS-CS
PO	4	12	0.2	0.2	19.44	19.45
POTV	4	12	0.2	0.2	21.29	21.31
DCT	4	12	0.3	0.3	32	32.54

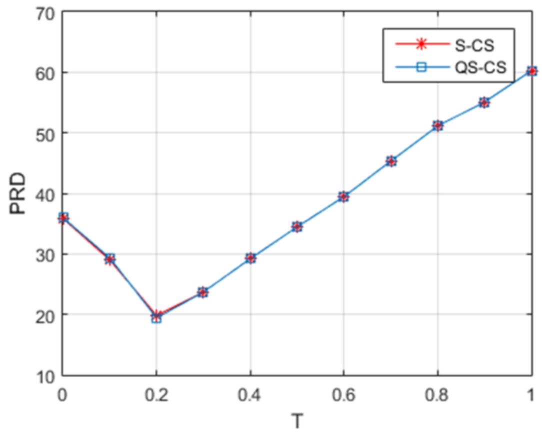

Figure 5. The fPCG signal reconstruction *PRD* of S-CS and QS-CS methods versus threshold, using PO space.

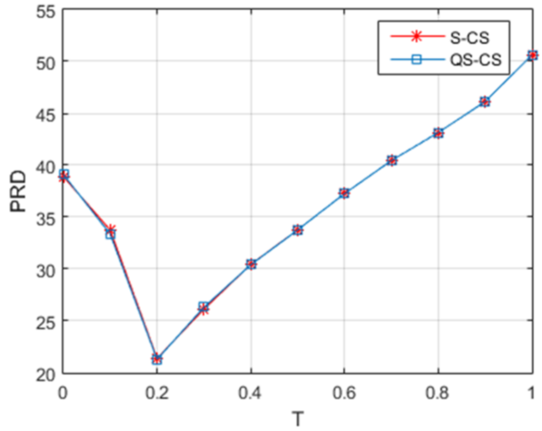

Figure 6. The fPCG signal reconstruction *PRD* of S-CS and QS-CS methods versus threshold, using POTV space.

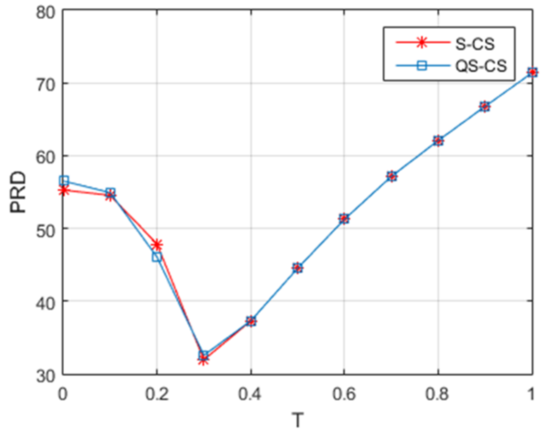

Figure 7. The fPCG signal reconstruction *PRD* of S-CS and QS-CS methods versus threshold, using DCT space

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