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Spectrum Leakage Effect Mitigation in RMPI Analog to Information Convertors

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Abstract- With emergence of Compressive Sensing (CS) theory, new signal acquisition devices called Analog to Information Convertors (AIC) were proposed. One of the most well-known AICs among various AIC architectures is Random Modulator Pre-Integrator (RMPI) AIC. Due to the window processing of signals in RMPI AICs, a phenomena called spectral leakage manifests itself in Discrete Fourier Transform representation of the signal. The spectral leakage in effect reduces signal sparsity and hence degrades quality of the recovered signal. Investigating this problem, in this paper we propose a modified architecture of RMPI analog to information convertor, which successfully handles spectral leakage issue through applying a window envelop in analog domain, prior to measuring the signal. Our simulation results suggest a 25 dB improvement in SNR of recovered signal.

Keywords- *Compressive Sensing, Analog to Information Convertor, Spectral Leakage, Analog to Digital Conversion*

I. INTRODUCTION

Analog to Digital convertors are considered as back-bones of modern electronic systems. However, ADCs are one of the most power hungry building blocks of the system, especially in high frequency regimes[1]. Recently some new analog signal acquisition devices based on Compressive Sensing theory are introduced[2]. These so called Analog to Information Convertors measure and recover the signal with acquiring minimum number of samples. In applications where the input signal is sparse, AICs potentially solve the power constraint imposed by ADCs. Some examples of these application are Cognitive Radio, Radar and Medical Imaging to name a few[2, 3].

Compressive Sensing signal acquisition framework relies on signal sparsity for exact recovery. There are some issues that might affect signal sparsity and high among them, is spectral leakage which severely reduces quality of the recovered signal and deteriorates the performance of the AIC[4].Paper [5] has investigated the recovery problem of frequency sparse signals in presence of basis mismatch. The paper provides a general analysis for sparsity basis mismatch and based on that, specifies an error bound for recovery and demonstrates poor performance of CS recovery in presence of leakage.

Although AICs can produce infinitely long sequence of measurements and hence avoid leakage problem, however the recovery algorithms can only deal with finite length of measurements. Therefore, the input signal must be measured and recovered in a segmented fashion. Windowing the signal (or literally limiting the signal in time) causes the discussed spectral leakage problem.

In this paper we tackle this problem through pre-processing the input in analog domain by application of window functions. We propose a modification in sampling procedure of RMPI AIC, through which a window function is applied prior to sampling. We show this modification reduces leakage and hence a higher quality of recovery is achieved.

II. BACKGROUND

A. Compressive Sensing

Compressive Sensing theory introduces a new framework for acquisition and recovery of sparse signals. Two fundamental assumptions of CS theory are signal sparsity and incoherent measurements. A N-length signal, X, is K-sparse in some domain like Fourier if there is only K non-zero coefficients representing signal in that domain. Assuming M measurements ($M << N$) are taken from signal through measurement system of:

$$\mathrm{Y} = \Phi \mathrm{X} \tag{1}$$

Where Φ is and $M \times N$ matrix which is sensing matrix. Based on theory if the components of Φ matrix chosen randomly the measurement with high probability would be incoherent[6]. The original signal is recovered through solving following problem:

$$\min_{x \epsilon R^N} \|\widehat{X}\|_1 \quad \text{subject to} \quad Y = \Phi \widehat{X} \tag{2}$$

In case, the signal, X, is sparse in some transform domain, using a proxy variable (α), the above expression can be re-written as below:

$$\min_{x \epsilon R^N} \|\widehat{\alpha}\|_1 \quad \text{subject to} \quad Y = \Phi \Psi \widehat{\alpha} \tag{3}$$

The above problem might be solve through convex optimization methods or greed algorithms[7].

B. RMPI Analog to Information Convertor

The main task of RMPI AIC is applying sensing matrix, Φ, to the input signal and taking measurements. For practical issues here sensing matrix is filled with random i.i.d Bernoulli (± 1) components. As**Error! Reference source not found.** depicts RMPI AIC is consist of some (here four) parallel channels. At any moment of time, each channel multiplies one of the rows of sensing matrix to the signal through mixers and quantizes the integration of resultant as a measurement. The digital recovery portion recovers signal from measurements vector, through performing CS recovery algorithms.

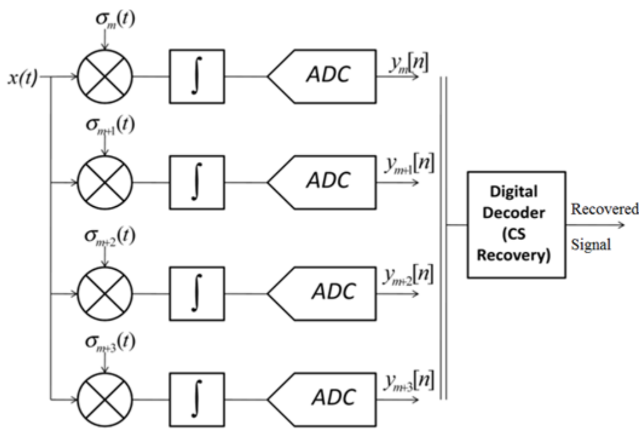

Figure 1. Architecture of RMPI AIC

In fig. 1, σ and y refer to elements of the sensing matrix and measurements vector respectively.

C. Spectral Leakage

A K-sparse frequency sparse signal can be written as sum of K complex sinusoids:

$$x[n] = \sum_{k=1}^{K} a_k e^{-jw_k n} \quad (4)$$

Where $w_k \in [0,2\pi]$ are frequencies of the sinusoids. A K-sparse representation of infinitely long signals of this type exists in Discrete Time Fourier Transform (DTFT) domain:

$$X(w) = \sum_{k=1}^{K} a_k \delta(w - w_k) \quad (5)$$

Where $\mathrm{sinc}(w) = \frac{\sin(\pi w)}{\pi w}$ and $l_k = \frac{N w_k}{2\pi}$.

Only when the all complex coefficients of the input signal are an integer multiple of analytic frequencies, the DTF representation is sparse.

$$f_{analysis}(m) = \frac{m f_s}{N}, \quad m = 0,1,2,\dots,N-1$$

Otherwise, their counterpart DFT coefficients will smear over all other frequency bins[4]. Hence, DFT representation does not maintain sparsity properties of DTFT representation. As stated in previous section, spectral leakage reduces signal sparsity and degrades quality of recovered signal in the CS framework. **Error! Reference source not found.** depicts performance of a typical AIC in terms of SNR of recovered signal for different types of inputs. The first type consists of complex frequencies components placed exactly on the DFT grid (best case), frequency components of the second type are chosen randomly (average case) and third case consists of frequencies places exactly between two grid points (worse case).

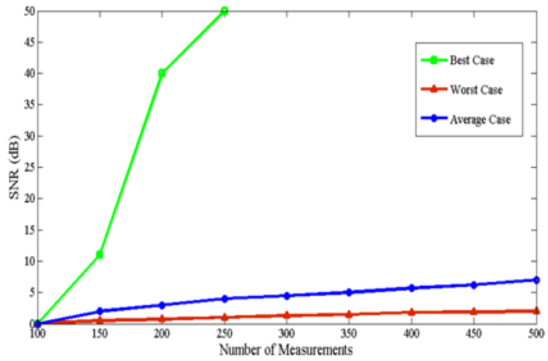

Figure 2. Comparison of CS recovery performance for different signal with different amounts of leakage

Recovery quality for the average case (the case we face in real applications) is far below the ideal best case, assumed in the theory.

III. PROPOSED METHOD

As mentioned earlier, we can use various window functions to reduce DFT leakage effect. Since every quantized measurements produced by RMPI is sum of a set of randomly weighted samples, applying a window envelope after quantization (in digital domain) is unattainable, though this is not the case with some other AIC architectures like Non-Uniform Sampler AIC where due to the its particular structure of sensing matrix it is possible to apply a window while performing CS recovery [8]. This is not the case with RMPI AIC architecture.

Therefore we propose a modified RMPI architecture in which a window function is applied on signal in analog domain and then CS based measurement recovery process is carried out. A proper windowing process must be adopted that reduces spectral leakage and prevents destruction of samples in the two edges of the windows as well. A windowing process breaks the signal into some possibly overlapping vectors. After processing, these vectors are stitch together through a synthesis function. A simple example of this process is the commonly used non-overlapping rectangular analysis and synthesis window function. To prevent loss of samples in window borders an overlapping windowing process must be adopted. We choose a 50% overlapping scheme and modified the structure of the convertor based on that.

As stated previously in RMPI AIC we have to apply window coefficients in analog domain. A switch-controller resistor or a variable gain amplifier might be used for multiplying window coefficients in the input signal. The proposed architecture is shown in **Error! Reference source not found.**.

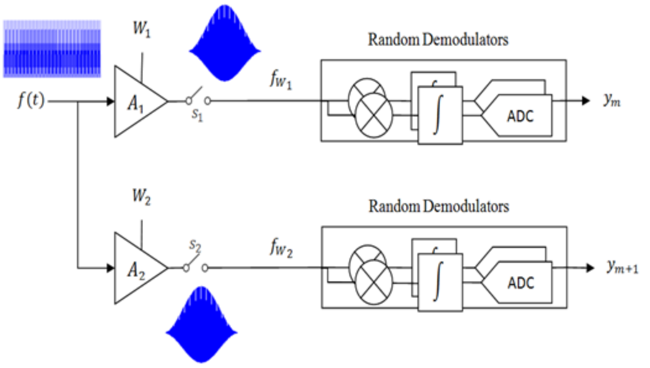

Figure 3. Proposed modified RMPI architecture

Window function of W_1 and W_2 are applied on the signal within a delayed interval of D samples (here $D = N/2$). In this architecture, channel of the RMPI AIC are divided into groups and form two collaborating separate RMPI AICs (e.g. a four-channel RMPI transforms into two two-channel AIC). The input signal is separately multiplied in two window functions (f_{w_1}, f_{w_2}) prior to sampling. Every group of channels forms their measurements vectors (y_1, y_2). Having the sensing matrix, each group reconstructs its own version of the windowed signals ($\hat{f}_1, \hat{f}_1$). These reconstruction signals are actually the digitized version of the windowed signal (f_{w_1}, f_{w_2}). To reconstruct the original signal, the synthesis function must be applied on ($\hat{f}_1, \hat{f}_2$) which is simply is achieved through summing them up.

Considering **Error! Reference source not found.** we explain the processing chain. Through analog multiplier, the input signal f(t) is continuously is multiplied to window functions.

$$f_{w_j}(t) = W_j\left(i + (j-1) \times \frac{N}{2}\right) \times f(t) \tag{6}$$

Where j represents the window index and W_j is the window coefficients vector. The windowed signals (f_{w_j}) are sequentially and continuously are generated, measured and recovered in CS framework.

$$y_1 = \Phi_{\frac{M}{2} \times N} \times f_{w_j} \tag{7}$$

$$y_2 = \Phi_{\frac{M}{2} \times N} \times f_{w_{j+1}} \tag{8}$$

Using the reconstruction algorithms (see section CS) $\hat{f}_{w_j}$ and $\hat{f}_{w_{j+1}}$ are independently recovered. Then, using synthesis function the original signal (the digitalized version of signal) is recovered. In 50% overlapping mode, all coefficients of the synthesis function are 1, and the signal is recovered as follows:

$$\hat{f}\left(i + \frac{jN}{2}\right) = W^s{}_j(i)\hat{f}_{w_j}(i) + W^s{}_{j+1}\left(i + (j-1) \times \frac{N}{2}\right)\hat{f}_{w_{j+1}}\left(i + (j-1) \times \frac{N}{2}\right) \tag{9}$$

Where $W^s{}_j$ and $W^s{}_{j+1}$ are synthesis functions and as said, here they both equal to 1.

IV. Simulation Results

In this section we present our simulation results. Experimentally, we find out hamming window function gets the best results and hence we chose Hamming window among all various window functions proposed to mitigate spectral leakage. In the first experiment, we compare reconstruction quality of a single sinusoid sampled through conventional and the modified RMPI AIC. The frequency of the sinusoid is varied from 1.8 MHz to 2 MHz. The window length is chosen to be 1000 ($N = 1000$) and 100 measurements are acquired through each model ($M = 100$). Simulation results are depicted in **Error! Reference source not found.**. The vertical axis show Signal to Noise Ratio (SNR) of the recovered signal as quality criteria.

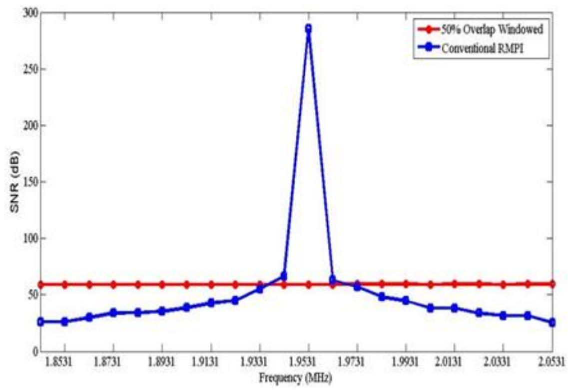

Figure 4. Comparison of recovery of a conventional RMPI and the proposed architecture

As it is seen in the **Error! Reference source not found.** reconstruction quality of the conventional RMPI is high only in vicinity of one of the integer frequencies ($\frac{m \times f_{sampling}}{N}$). However, the modified RMPI with 50% overlapped windowing scheme, tends to recover signal with higher SNR, independent of the input frequency.

There is a problem associated with the proposed structure and it the division of the channels into two separate groups. In effects, this leads to acquiring fewer measurements for every window and hence potentially worsens the situation. Experimentally we figured out that a larger window size produces better results in the modified architecture but not as ell in the conventional.

Figure 5 shows the reconstruction quality for window sizes of 512, 1024 and 2048. The input signal consists of 20 complex sinusoids at random frequencies.

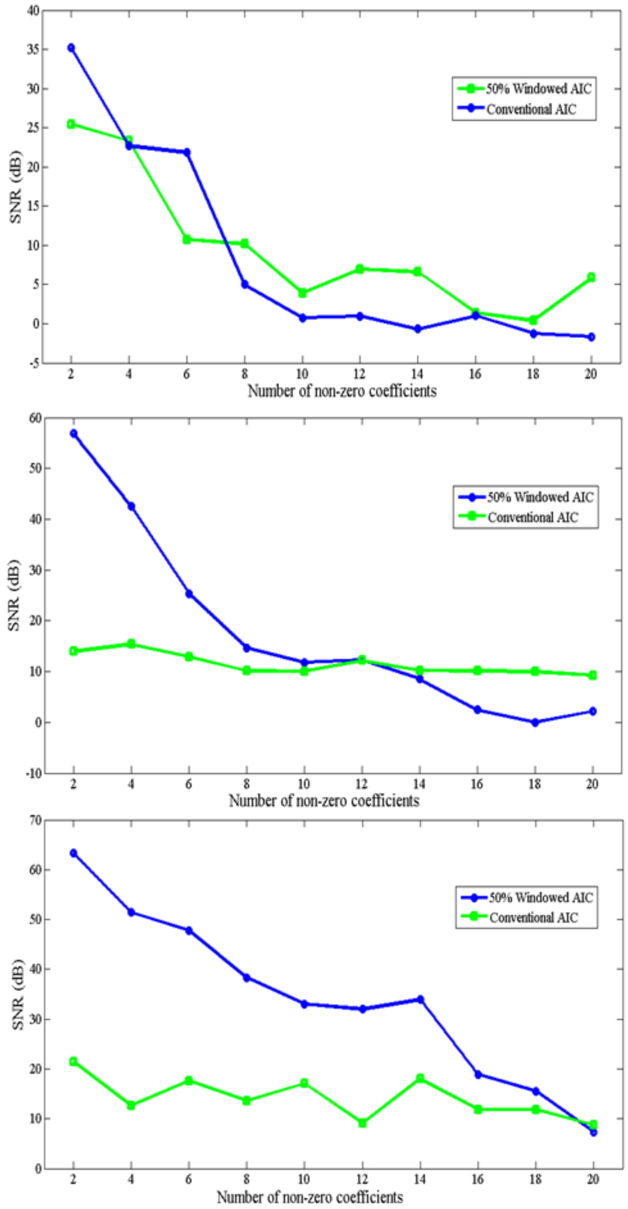

Figure 5. Effect of window length on performance of the proposed method

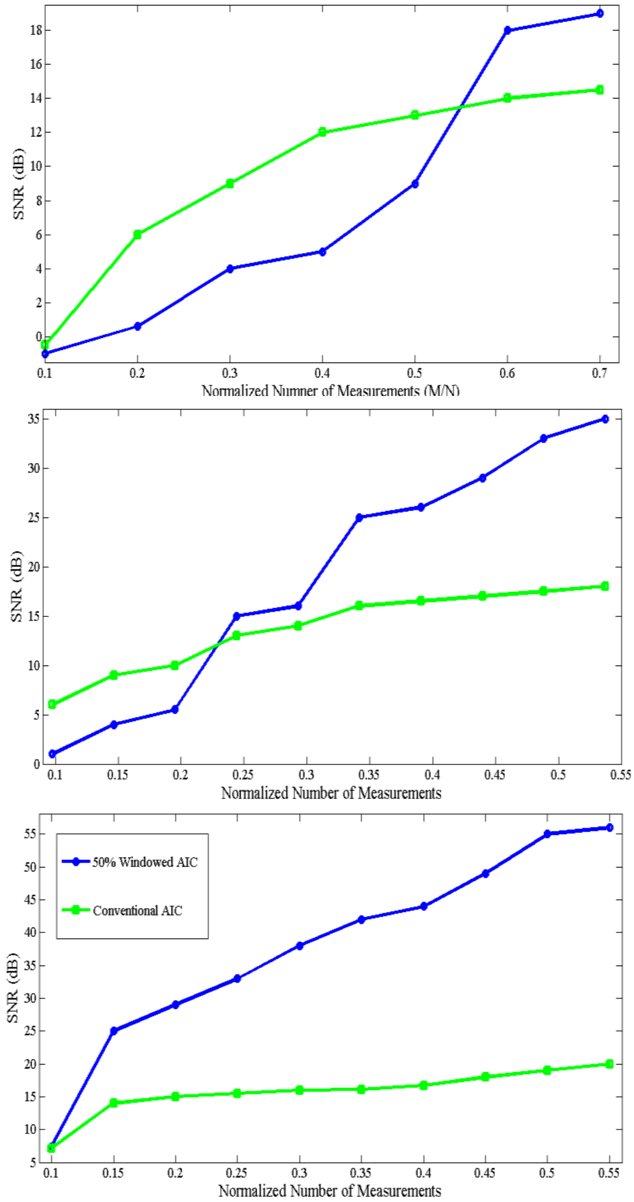

Figure 6. Effect of sparsity level on performance of the proposed method

In each picture we marked the point where our method becomes superior to the conventional architecture. As it is observed, when the window size is large (e.g. its 2048) the proposed method is always superior to the conventional method. Although the quality performance of conventional method increases with higher window length, however it seems that the conventional method make a poor use of larger window lengths.

In the next experiment, we varied level of sparsity of the input signal from 2 complex sinusoids to 20 complex sinusoids and measured and recovered signal through conventional and proposed RMPI models. In this experiments again different window lengths were used (512, 1024 and 2048) but normalized number of measurements kept fixed ($M/N = 0.2$). The results are depicted in fig. 6.

V. CONCLUSION AND FURTHER WORKS

One of the issues that limit performance of RMPI AICs is spectral leakage. In this paper, through modifying architecture of RMPI AIC and using window functions, we propose a method which significantly decreased spectral leakage effect on recovery. In the proposed method, a window functions applied to signal prior to sampling through RMPI. The proposed method increases SNR of recovered signal by 25 dB, but a proper window size must be selected to achieve this gain. The modified architecture acquires fewer measurements for every window (technically half of the total capacity is devoted to every window). This might be solved through adapting a 25% overlapped windowing process and using more complex synthesis functions.

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