International Journal of
Science and Engineering Investigations vol. 5, issue 58, November 2016
ISSN: 2251-8843

Investigation of Seismic Stability of Heterogeneous Slopes

Enayatollah Khoubani[1], Arash Toutounchi[2]
[1]M.Sc., Department of Civil Engineering- Geotechnic, Yasouj Branch, Islamic Azad University, Yasouj, Iran
[2] Professor, Department of Civil Engineering, Yasouj Branch, Islamic Azad University, Yasouj, Iran
([1]anayatkhoubani@yahoo.com, [2]Isiportal2005@gmail.com)

Abstract- This research is based on the theory of linear lower bound limit analysis and finite element method and its main topic is calculation of safety factor of heterogeneous slopes in case of earthquake. The results of proposed method along with upper bond response limit the safety factors in a range of 10 percent or lower. This could help to provide a relatively accurate and reliable response, and also assess the answer that could be obtained from other methods. When the load of foundation is low, critical condition related to the slope instability, and sliding mechanism has two-dimensional nature. When the load of foundation is high, instability related to inadequate bearing capacity of the foundation adjacent the slope. In this case, there is a three-dimensional problem and the actual safety factor is obtained by performing a three-dimensional analysis.

Keywords- *limit analysis, lower bound, linear finite element, slope, bearing capacity*

I. INTRODUCTION

The problem of slope stability analysis is one of the most important and yet most complex issues of soil mechanics, and close examination of it will not be possible without a profound appreciation of soil mechanics. Limit equilibrium method are very common for analyzing slope stability. In this method, at first a hypothetical failure mechanism is considered and corresponding safety factor is calculated, then by trial and error the weakest sliding surface, corresponding to the minimum safety factor, is obtained.

Methods used in stability analyzes are mostly in one of the following groups:

1. Limit equilibrium methods or direct methods, based on rigid plastic soil behavior.
2. Numerical methods or indirect methods based on elastic plastic soil behavior.

Limit methods, including methods of equilibrium, characteristic lines and limit analysis, are based on relatively simple model of soil behavior, and their main advantage is to find closed form common solutions for practical problems. With these methods it is possible to study different parameters like strength and geometry of soils.

In contrast, in the indirect methods all equilibrium, compatibility and behavior equations, have been satisfied and it is possible to use sophisticated behavioral relationships, and also deformations before failure can be calculated, but additional calculations and assessing of parameters are very time consuming.

Duncan (1996) pointed out that the finite element method (FEM) is a common method which can be used to calculate the stresses, deformations, pore pressure and other variables of soil materials without other assumptions.

Potts et al. (1997) used the finite element method to calculate vertical slope failure mechanism in stiff clay soils. To predict slope stability using finite element method and obtain the safety factor, strength reduction method (SRM) is used extensively. Griffiths and Lane (1989) and Griffiths and Marquez (1988) used the method for analyzing 2D and 3D slopes. They concluded that the use of strength reduction method (SRM) provides very reliable results. Huang et al. (2002) observed that the critical slip surface that is calculated by the method of simplified Bishop, well corresponded with results that were concluded from finite element method. In addition, the difference between SRAM and limit equilibrium methods of calculation is very small (Baker et al (2001) and Psarropoulos & Tsomponakis (2008)). The past researches have pointed out that Griffiths and Marquez only studies the three-dimensional finite element method. They used this method to study the influence of geometrical parameters and site topography on Canyon dam. Azzouz and Baligh (1975) assessed effects of sides of the sliding mass in cohesive soils. Tsai and Huange (2000) examined bending force equations as well as equations in three dimensions the potential for mass disruption. Gens et al. (1988) have been presented a series of 3D charts for slopes with cohesive soil. They concluded that between 2-D and 3-D results, there is a difference of about 3 to 30 percent. They calculated the average value of the difference between 2D and 3D analyzes of % 13.9. The difference has not trusted in condition where slope is made of cohesive undrained soil and reverse analysis is done. Yamagami et al. (2006) followed a series of studies that applied figures were derived from the results of their work. The study was conducted on cohesive soils with internal friction. They found that unlimited slopes are limited safety factor than the simple slopes. Baker et al. (2006) used pseudo-static method (PS) for analyzing slope limit equilibrium in 2-D cases. Their study was concentrated on the effects of Pseudo-static method on stability of slopes specified for a range of geometric quantities, internal friction and stability number $(N = \frac{c'}{\gamma H})$. This form of the number of stability is similar to hypotheses have been considered previously by Gen (1988) and Taylor (1937). Hamedan and Chem (1982) have presented a three-dimensional limit equilibrium method and pointed out that in the 3-D mode the safety factor is less than 2-D mode. After them, Cavendish (1987) proved that Chen and Chameav result was wrong. He also noted that for a specified slope, the safety factor in 3-D mode always is higher than 2-D mode. Stork Eid (1998) were tested 3 samples of commercial software applications, were developed based on the limit equilibrium methods and modeled several landslides. They concluded that output safety factors of these software applications were very low. The reasons that these software applications could not calculate appropriate safety factors comprise of limitations for defining the geometry of the problem, materials and ignoring shear strength of vertical failed mass. Although the limit analysis theory is a very simple and useful method to analyze the Geotechnical formation, but application of this theory is seldom done in three-dimensional analysis issues, and most research has been used 3D limit analysis theory according to upper boundaries. Including these studies Chen et al. (2001, 2003), Doland and Chen (1997), Farzane and Askari (2003), Debuhan (1998), Michalowski (2002,) and Viratjondr & Michalowski (2006) can be cited. Due to the difficulties to create manually an acceptable stress field, Limit analysis method in most prior research focused on the upper bond boundaries and despite the fact that the upper boundaries method could be a very good estimation to answer questions, but because of the fact that lower boundaries are safer, so they are used more in practical problems. Solution based on the lower bound is subject to the satisfaction of the boundary conditions and stress discontinuity as well as the equilibrium equations and yielding conditions, and all of this is apply as a series of constraints on the model of the stresses. The solution were presented by Yamin and Sloan (2002) for the first time, and developed by Totonchi and Askari (2011). Lyamine and Sloan (2002) provided a new method of using lower bound and finite element method. The lower boundary has been established based on acceptable tension fields. The final objective of this study is to develop methods and solutions of Tootoonchi and Askari (2011) in 3D seismic impact loading for flat slopes.

II. ANALYSIS METHOD

A combination of finite element method and lower bound theory is developed in this study. This means that considered soil slopes under the three-dimensional modeling based on the finite element method and lower boundary conditions which consist of equilibrium equations, discontinuity conditions of stresses, boundary conditions and yielding condition are considered as constraints of the problem. In order to solve this problem a stress field with highest safety factor is considered (the highest lower bound safety factor).

Therefore, in addition to the finite element method (modeling) and theoretical lower bound (solving) an optimization code is used to calculate the maximum the lower bound. The models in this program will be three-dimensional models; therefore preferred method for optimization is important. Due to the rapid and proper solution of nonlinear optimization method and the presence of a default application in MatLab environment, this optimization method is chosen for evaluating the maximum lower bound of the safety factor. To ensure proper responses for lower bound method, this technique is a good path to assess the stability of sample slopes. Responses that were extracted from the mentioned methods will be presented through a series of dimensionless diagrams. These diagrams are a very convenient tool for engineers to enable them to obtain the safety factors of man-made embankments or walls of the excavation. Although often traditional limit equilibrium methods used to calculate the slope stability in 2 and 3-dimensional, but somewhat the accuracy of this method is still under question. This is due to assumptions that are considered in solving problems. There are very strong reasons for using linear element method. The reasons are usually consistent with lower boundary conditions, including:

1. By making linear changes among the stresses, if the yielding conditions satisfy in all points, surely the yielding condition will be established among the points. Therefore, yielding conditions constrains apply only on the points, and the number of inequality constraints is reduced considerably.

2. Using linear finite element method, introduction of stress discontinuities are relatively more easily until the pages are just curves.

3. Using linear finite element, boundary areas that are affected by external load, are modeled easily. Whereas in the area of applied load on the boundary curve, a number of linear elements can be placed side by side with an appropriate approximation, and estimate the curved surface.

4. In the final optimization to reaching the highest lower bound, only linear equations are produced, that will be helpful for reducing analysis time.

Because of the complexity of the issues and problems in the three-dimensional optimization methods, test method that can be substituted for other analytical methods, without development of appropriate software, is impossible. Therefore, in the first phase it is essential to provide software with the necessary capabilities. This software is called FELAB (Finite Element Method + Limit Analysis + Lower Bound), was prepared and used to continue the research. The FELAB code models the slopes using the finite element method and detailed element, then equality and inequality constraints due to the lower bound conditions (equilibrium, stress discontinuity and boundary condition equations), and yielding condition are defined, and after that with regard to the objective function, the maximum value of slope safety factor in the lower bound condition, is obtained. With the completion of the analysis, results are presented in a vector that contains an acceptable stress field. By putting the stress on yield criterion it is determined that a group of points are in plastic and others are in elastic state. Applications of this method in various issues show that, the results of this study with the results of the upper bound (Farzaneh and Askari (2008)) to create a range with accurate answers. According to the presentation of new meshing method that is used in this study, there is capability of modeling slopes that have changes in their plan, and none of the pervious methods capable of modeling on the basis of the lower bound theory. One way to examine the methods of three-dimensional is evaluation of two-dimensional solutions of them in particular. If the width of mechanism used in the methods of three-dimensional compare to height ratio of slope is large, it actually will be two-dimensional and obtained results, is supposed to conform to the existing two-dimensional analysis results. In Fig. 1 comparison between the results FELAB and results of equilibrium (modified Bishop) is performed. This comparison is for slope of 30 degree ($\frac{\gamma H}{c}\tan\phi = 2$ /λ= 30) in this situation value of 2D stability number (Ns= γH/c) calculate with limit equilibrium method Ns (2D) =12.9. As is evident from this figure, stability number decrease by increasing the value of L / H ratio. This value (L/H) is 1.17 times of 2D value for the lower bound that is obtained by limit equilibrium method. This value decrease with increasing of L/H ratio, where the value became less than 2D stability value (Ns (2D)). It should be noted that the responses of FELAB code is based on 36 elements. This increasing of elements led to better and more accurate responses. In Figure (5-10) comparison between the results of FELAB by changing the number of elements is provided.

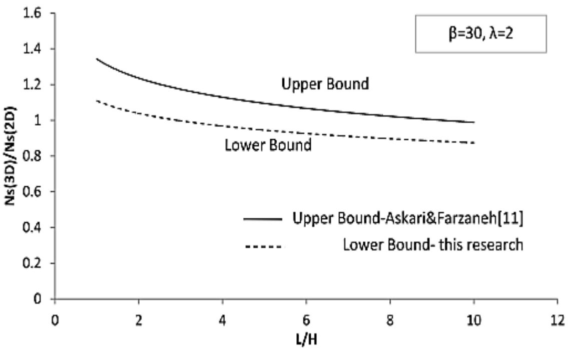

Figure 1. Comparison of two-dimensional limit equilibrium model responses with flat models in both the upper and lower bound situations.

Parameter $\lambda_{\varphi c}$ = λ in this graph is defined as follows:

$$\lambda_{\varphi c} = \gamma.H \ / C.\tan(\phi) \quad (1)$$

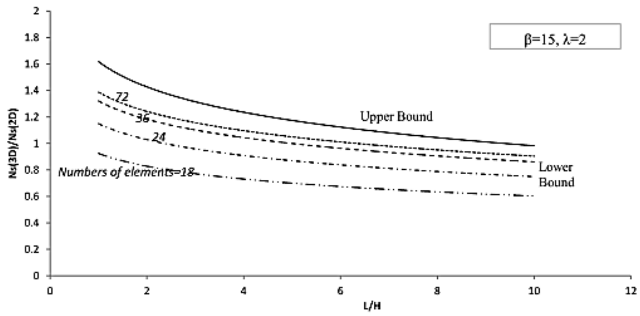

Figure 2. Comparison of lower bound responses with the change in the number of elements of flat slope model (Tootonchi & Askari 2011).

Fig.1 and Fig.2 presented similar analysis of upper bound problem. These results have been presented by Tootonchi & Askari (2011) in 3D state. As it is obvious, responses of the three-dimensional lower bound and upper bound are in good agreement with each other. These results have the potential to represent a range of different values of L / H ratio which the exact response would certainly place between these ranges. On average, this range limits the exact responses about 9 percent higher or lower.

One of the most common methods of two-dimensional stability analysis of homogeneous slopes is using Stability Charts. These diagrams are presented mainly based on the limit equilibrium methods and would give stability numbers for the different slopes. Stability number (N_m) is reverse of stability coefficient (N_s) . For instance, presented graphs by Taylor (1937). In Fig. 3a three-dimensional analysis of Ugai (1985), Leshchinsky (1986) and Farzaneh & Askari (2008), have been compared with the results of the present analysis. As it can be seen, FELAB code results are very satisfactory. These results have been obtained from 72 element models. With comparison of Bishop Method and current study, it is known that at low L/H ratios, despite lower bound responses of this method, results are presented higher values than 2D Bishop Method. Fig. 3b presents comparison of these results.

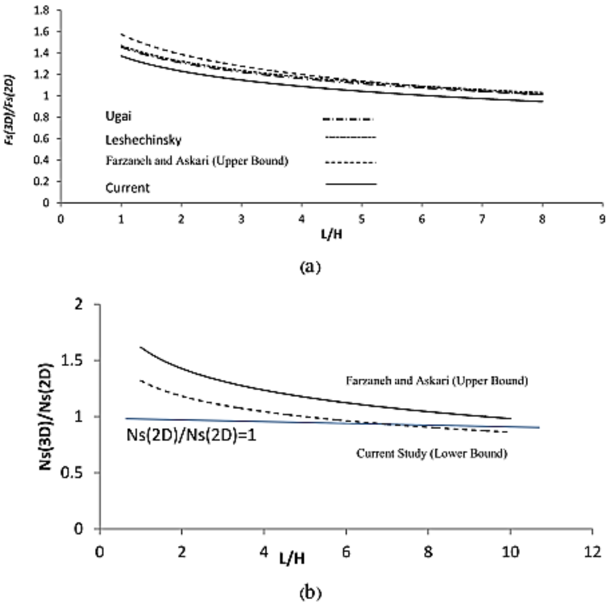

Figure 3. a. Comparison of coefficient safety factor of 3D with 2D in Ugai, Lishiniski, Farzaneh & Askari (Upper bound) and current study analysis, b. Comparison of Bishop method and current study.

One of the important parts of modeling is element determination in the models. Certainly, increasing of elements led to more accuracy and validity of responses. But, the more model elements, the more time to run the code. Therefore, in this part some models with different elements have been compared. In Fig. 4 for the constant values of $\lambda\varphi c = 2, \beta = 30$ and elements of $En = 18, 24, 36, 72$, the results compare together.

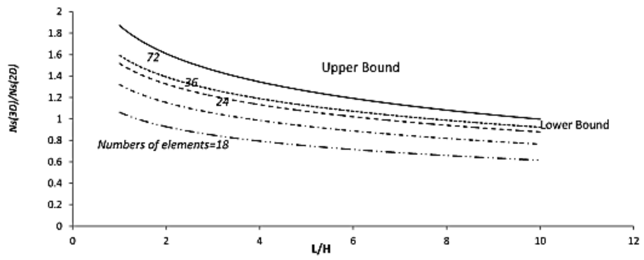

Figure 4. Comparison of analysis result of slope with different elements.

In Fig. 4 β is slope inclination and En are number of elements in each model. As it can be seen, as the number of elements increased, the space between upper and lower bound decreases. This shows increasing in accuracy of responses, but the rate of lower bound response changes, decreases. Consequently, for higher values of elements, changes of lower bound response are negligible. In this study, because of decreasing of the rate of response changes (in comparison with more than 36 elements), and also more speed in responding, 36 elements has been used. To compare responses and establish the validity of the proposed method, the results have been compared with the results of other researchers. The results are in good agreement with the majority of other researchers methods. Fig. 5 present $\frac{F_{3D}}{F_{2D}}$ ratio (which F_{iD} is safety factor in the iD dimension) against the variation of L/H. As can be seen, the result of this research has been accurately.

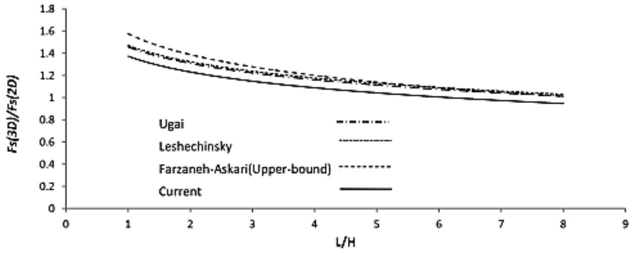

Figure 5. Comparison of current study with Ugai ،Leshcheneski ،Baker ، Askari and Farzaneh.

One of the uses of slope stability analyzes is to investigate the slope in circumstances where a new foundation will be built upstream. In Fig.6 results of FELAB code (variation of F_3 safety factor for different values of q in terms of L changing) was studied in particular case. As can be seen, in cases where the value of q is equal to 20 or 100 kPa, the safety factor is reduced with increasing of L / B ratio, that B represents the foundation width. These results have been obtained from lower bound analysis of a slope with a 20 degree angle of friction, cohesion of 20 kPa, specific gravity of 20 kN per cubic meter and under loading condition, as is presented in Fig. 6.

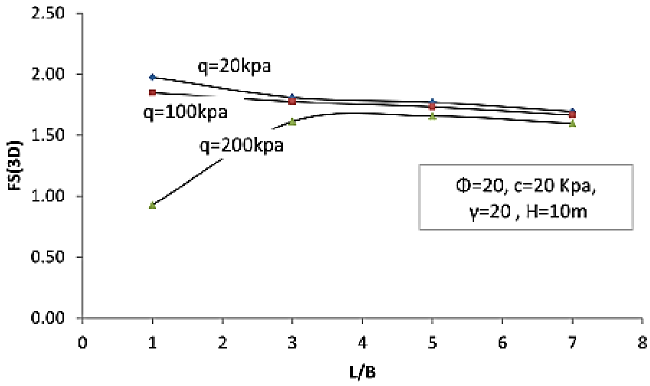

Figure 6. Comparison of safety factor for lower bound condition and 3D analysis under different load conditions.

In this section, for two loads of 20 and 100 kPa, sliding mechanism is corresponding with a minimum safety factor of two-dimensional mechanism. On the other hand, in case that q is equal to 200 kPa, the safety factor increase with increasing of L / B ratio, it means that, the minimum safety factor is related to three-dimensional mechanisms (Fig. 7).

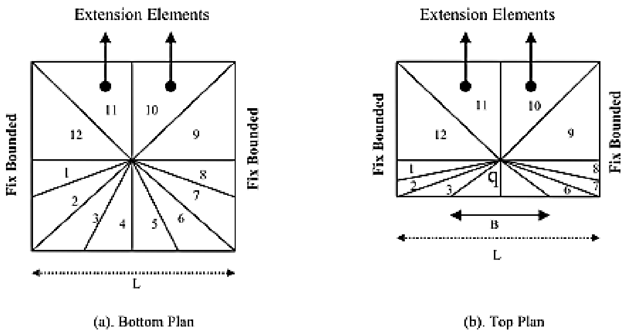

Figure 7. Method of loading in FELAB code.

Thus, there are two very different modes for stability analysis of foundation located in the vicinity of a slope:

1. In cases where the foundation load is low, critical condition is related to slope instability and sliding mechanism has two-dimensional nature. In this case, effect of foundation load is negligible in calculations, and consequently, the safety factor value obtain with two-dimensional analysis that q value is assumes zero.
2. In cases where the foundation load is high, instability is related to inadequate bearing capacity of the foundation in the vicinity of slope. In this case, the issue is essentially three-dimensional and real safety factor is obtained by performing a three-dimensional analysis.

It should be noted that in addition to the foundation load, soil shear strength parameters and geometry are effective in determining the critical conditions of instability. However, in the two-dimensional analysis, the possibility of separating the two mentioned above is not possible. Also in these methods, where there is external load, point loads and single foundation is considered linear and the problem is analyzed and modeled in 2D mode. These kinds of analysis are on the safe side, but do not get an accurate picture of the critical conditions.

Analysis of Chen et al (2001) were established in two layers soils. In this analysis which has been created by limit analysis and spiral log of sliding mechanism, some slopes with geometry shown in Fig. 8 have been studied

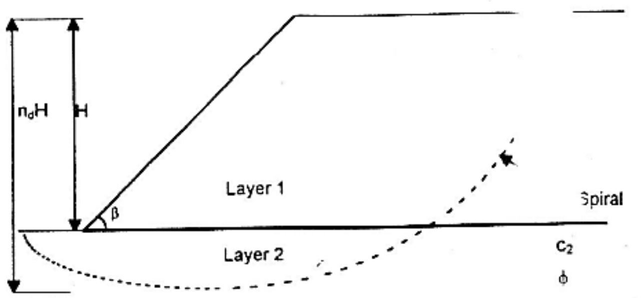

Figure 8. Cross section of analyzed heterogeneous slope by Chen et al (2001).

In Chen's research a slope established at angle β and cohesion of C1, and bottom layer starts from the foot of the slope. Lower layer cohesion was C2 and internal friction for both layers is assumed to be equal. FELAB code was used to investigate the slope stability with two layers and $c_2/c_1=0.1$, similar to Chen's and Askari's (Fig. 9). Fig. 10 shows two kinds of slopes in FELAB code. Fig. 11 presents results of stability analysis for Φ= 5, 15, 25 with 5 m height and 15 kN/m^3 specific gravity, in the lower bound case and changing of L/H ratio (L is length of slope).

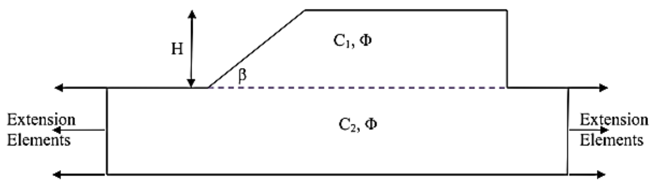

Figure 9. Stability assessment of multilayer slopes with the same features and $c_2/c_1=0.1$ ratio.

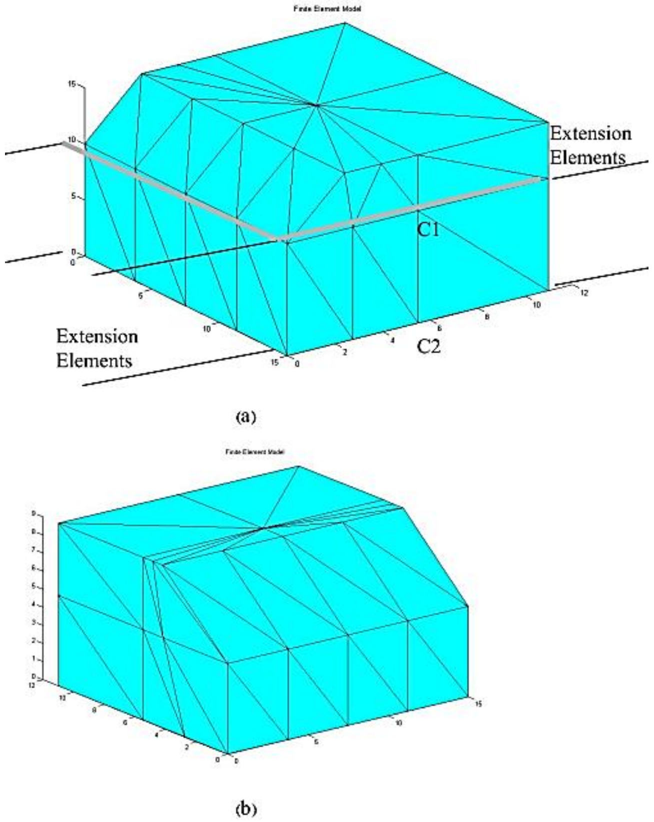

Figure 10. Two samples of slope in FELAB.

It is obvious that, as shear strength of bottom layer increase, penetration depth of critical surface decrease in this layer, consequently, depth coefficient decrease.

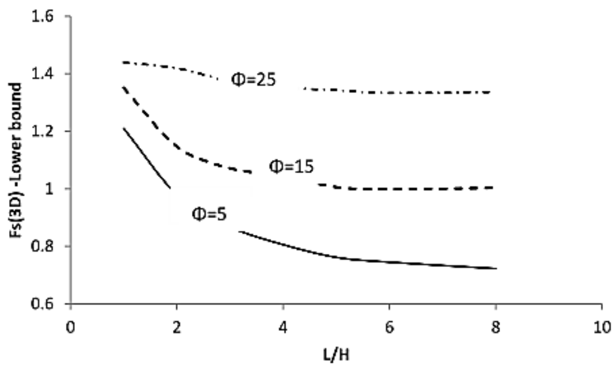

Figure 11. Stability results of heterogeneous slope with Φ = 5, 15, 25 and c_2/c_1=0.1.

Results of these models have been compared with limit equilibrium method in 2D mode. Table 1 shows the variation of safety factors in 2D mode.

TABLE I. SAFETY FACTORS FOR 2D MODE WITH LIMIT EQUILIBRIUM METHOD.

β	Φ	Fs(2D)
40	5	0.738
40	15	1.063
40	25	1.487

Comparing limit equilibrium method results with lower bound 3D in the case of L/H=8 (Fig. 12) show that, there is 5 % difference between 2D and 3D mode. A 40 degree slope with the height of 5 m, has been shown in figure 21.6, was modeled with FELAB. Soil layer characteristics are:

γ_1= 21 KN/m^3 C_1= 20kpa φ_1= 5

γ_2= 19 KN/m^3 C_2= 10kpa φ_2= 30

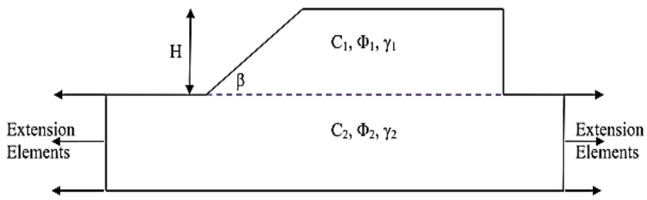

Figure 12. FELAB ability to analysis heterogeneous slopes.

Fig. 13 shows variations of 3D to 2D safety factors (F3/F2) in terms of L/H.

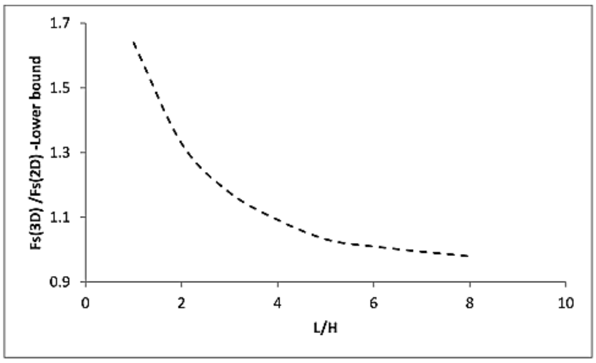

Figure 13. Variations of 3D to 2D safety factor in the slope shown in Fig.6-51 (F3/F2).

Above analysis for a 60 degree slope and assuming two equal layers, leads to the following graphs (Fig. 14 and Fig. 15). Underlying layer specifications is considered to be constant.

γ_2= 19 KN/m^3 C_2= 10kpa φ_2= 30

Similarly, corresponding to the above graphs (Fig. 14 and Fig. 15), for horizontal seismic acceleration coefficient = 0.1, 0.2, and 0.3 kh, charts are as follows (Fig. 16, Fig. 17 and Fig. 18),

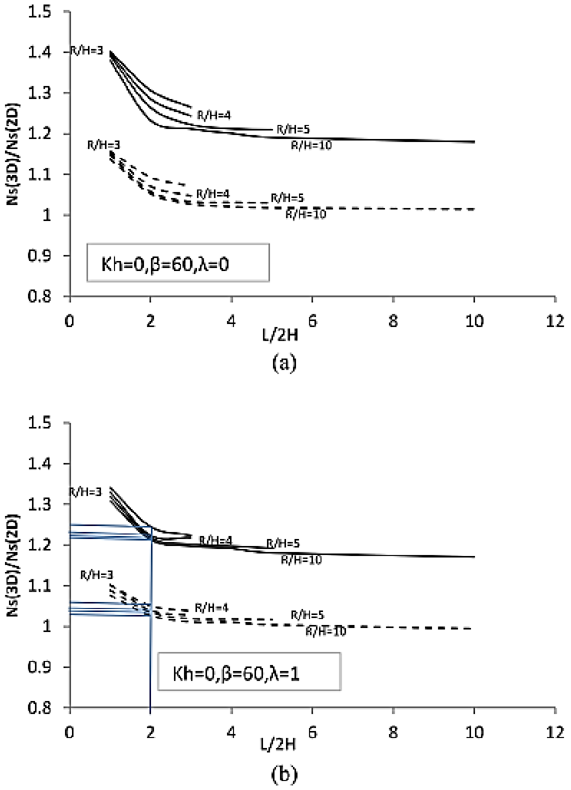

Figure 14. Stability graphs for convex slopes with following specifications β=60, (a). λ=0; (b). λ=1; (c). λ=3; (d). λ=10

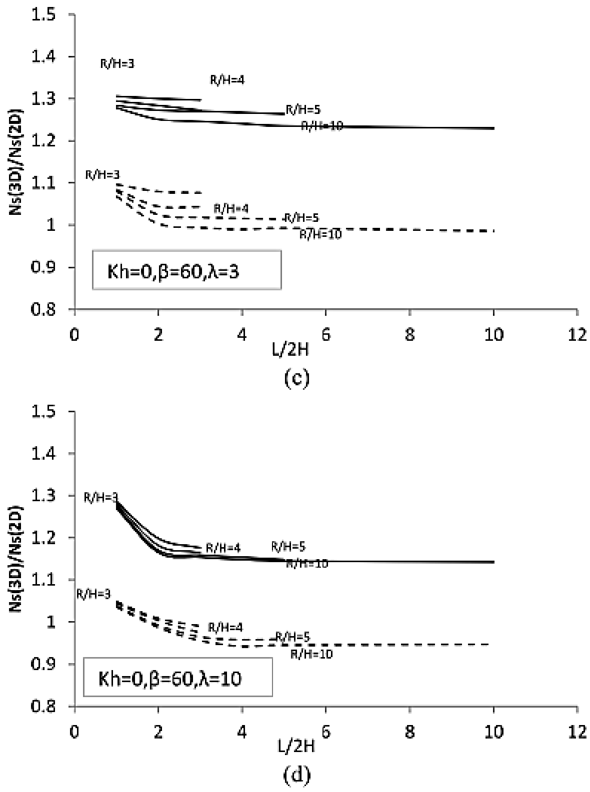

Figure 15. Stability graphs for convex slopes with following specifications β=60, (c). λ=3; (d). λ=10

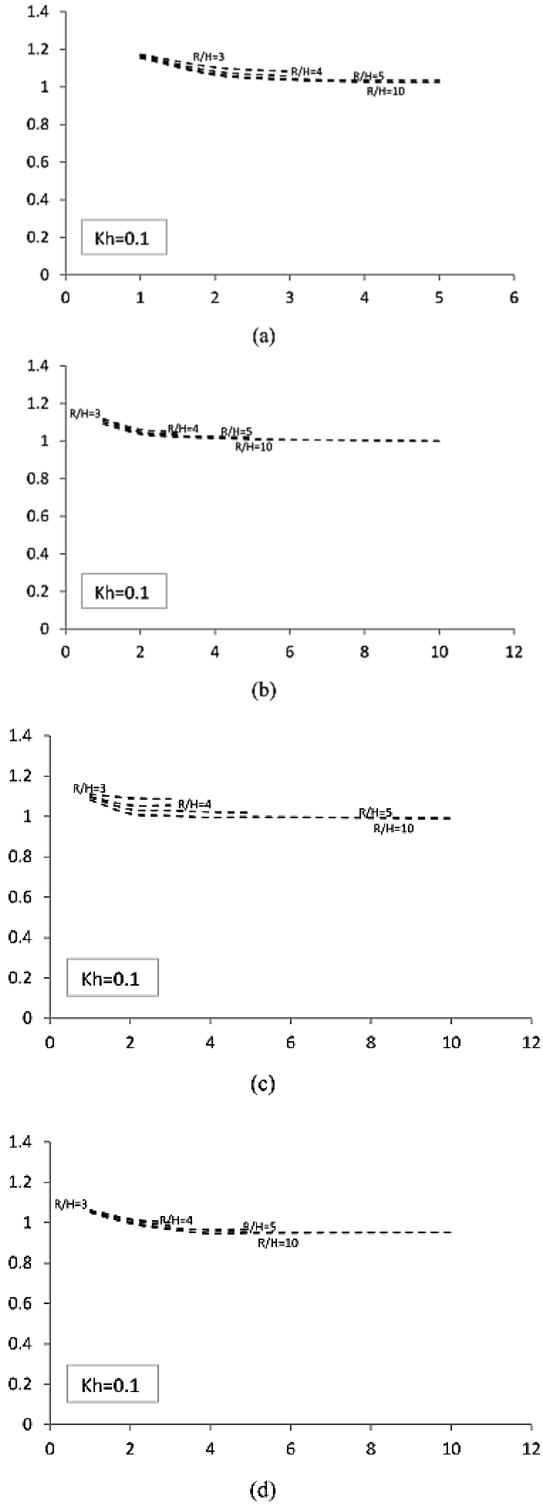

Figure 16. Dynamic stability graphs for convex slopes with following specifications: β=60, Kh=0.1, (a). λ=0; (b). λ=1; (c). λ=3; (d). λ=10

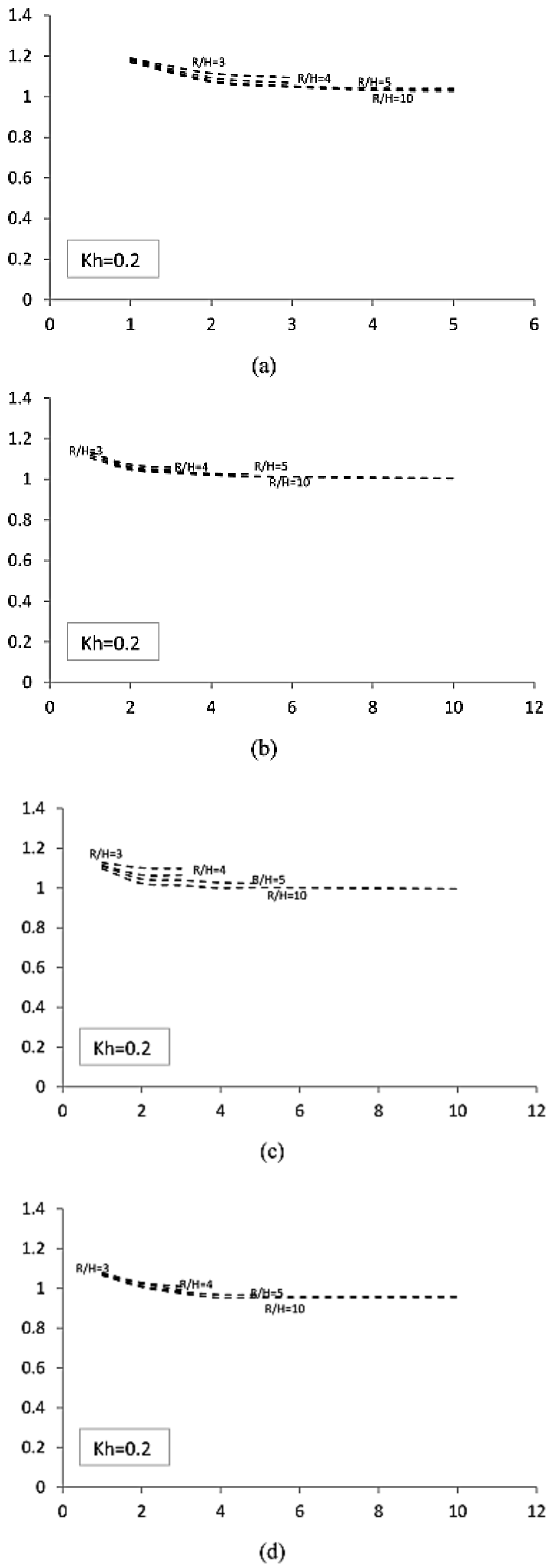

Figure 17. Dynamic stability graphs for convex slopes with following specifications: β=60, Kh=0.2, (a). λ=0; (b). λ=1; (c). λ=3; (d). λ=10

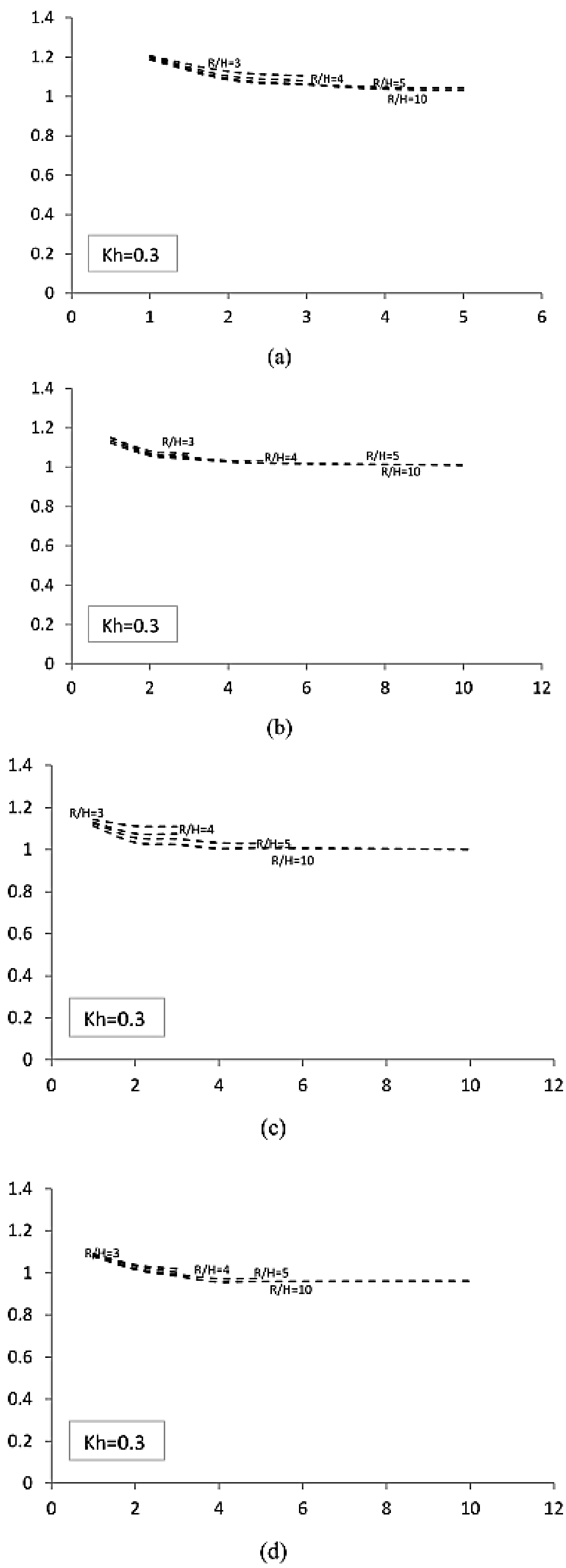

Figure 18. Dynamic stability graphs for convex slopes with following specifications: β=60, Kh=0.3, (a). λ=0; (b). λ=1; (c). λ=3; (d). λ=10

III. CONCLUSION

This research is based on the theory of linear lower bound limit analysis and finite element method and its main topic is calculation of safety factor of heterogeneous slopes in case of earthquake. Based on the findings of this research, the following conclusions are made.

- The results of the proposed method along with upper bound responses, limit the safety factor in a range of ±10 percent. Therefore, it can make fairly accurate responses and also, assess the obtained responses from other methods. As it is showed in the graphs, F_{3D}/F_{2D} ratio increase by decreasing β ،Kh and L/H.
- The stability number N_s decrease by increasing β ،Kh and,L/H and if L/H ratio move towards infinity, for a certain amount of β , Ns value reach to its minimum. Therefore, it shows decreasing of safety factor with increasing of L/H.
- FELAB code also has the ability to investigate the stability of heterogeneous slopes. The slope of the subsurface layers can be considered non-horizontal and there is no limitation on the number of layers.
- In the FELAB code, similar to the upper bound method, there is no limitation on the investigation of different angle of internal friction for adjacent layers.
- Using the presented example, 3-dimensional analysis results of slope for 4 modes of L/H=1, 2, 3, 4 is about 1 to 1.16 times of 2-D analysis.
- According to the presented graphs, it is obvious that the more cohesive soils, the more ،Fs(3D)/ Fs(2D) ratio.

REFERENCES

[1] Baker R, Shukha R, Operstein V, Frydman S, "Stability charts for pseudo-static slope stability analysis, Soils Dynam Earthquake Eng. 2006; 26:813–23.

[2] Baligh, M. M., and Azzouz, A. S, "End effects on stability of cohesive slopes, J. Geotech. Engrg. Div. 1995, 101-11, 1105–1117.

[3] Cavounidis S, "On the ratio of factors of safety in slope stability analyses, Geotechnique 1987; 37(2):207–10.

[4] Chang M. A, "3D slope stability analysis method assuming parallel lines of intersection and differential straining of block contacts, Can Geotech J.

[5] Chen RH, Chameau J-L, "Three-dimensional limit equilibrium analysis of slopes, Geotechnique 1982; 32(1):31–40.

[6] Chen Z, Wang X, Haberfield C, Yin J-H, Wang Y, "A three-dimensional slope stability analysis method using the upper bound theorem part I: theory

[7] De Buhan P, Garnier D, "Three dimensional bearing capacity analysis of a foundation near a slope, Soils Found 1998; 38(3):153–63.

[8] Donald IB, Chen ZY, "Slope stability analysis by the upper bound approach: fundamentals and methods, Can Geotech J 1997; 34:853–62.

[9] Duncan JM, "State of the art: limit equilibrium and finite-element analysis ofslopes, J Geotech Eng ASCE 1996; 129(2):577–96.

[10] Farzaneh O, Askari F, "Three-dimensional analysis of nonhomogeneous slopes, JGeotech Geoenviron Eng ASCE 2003; 129(2):137–45.

[11] Farzaneh, O., and Askari, F., Ganjian, N, "Three Dimensional Stability Analysis of Convex Slopes in Plan View, J.Geotech and Geoenviromental Eng. ASCE, 2008; 1192-1200

[12] Farzaneh, O., and Askari, F, 3D stability of slopes using upper bound limit analysis method, PhD thesis, Tehran University, 1994.

[13] Gens a, Hutchinson JN, Cavounidis S, "Three-dimensional analysis of slides in cohesive soils, Geotechnique 1988; 38(1):1–23.

[14] Griffiths DV, Lane PA, "Slope stability analysis by finite elements, Geotechnique1999; 49(3):387–403.

[15] Griffiths DV, Marquez RM, "Three-dimensional slope stability analysis byelasto-plastic finite elements, Geotechnique 2007; 57(6):537–46.

[16] Hovland HJ, "Three-dimensional slope stability analysis method, J. Geotech EngDiv ASCE 1977; 103(9):971–86.

[17] Huang C-C, Tsai C-C, "New method for 3D and asymmetrical slope stability analysis, J Geotech Geoenviron Eng ASCE 2000; 126(10):917–27.

[18] Hwang J, Dewoolkar M, Ko HY, "Stability analysis of two-dimensional excavated, Can Geotech J 2002; 39:1026–38.

[19] Jiang J-C, Yamagami T, "Charts for estimating strength parameters from slips inhomogeneous slopes, Computer Geotech 2006; 33:294–304.

[20] Lane PA, Griffiths DV, "Assessment of stability of slopes under drawdown conditions, J. Geotech Geoenviron Eng 2000; 126(5):443–50.

[21] Lyamin AV, Sloan SW, "Lower bound limit analysis using non-linear programming, Int J Numer Methods Eng 2002; 55:573–611.

[22] Michalowski RL, "Three-dimensional analysis of locally loaded slopes, Geotechnique 1989; 39(1):27–38.

[23] Michalowski RL, "Stability charts for uniform slopes, J Geotech Geoenviron EngASCE 2002; 128(4):351–5.

[24] Potts DM, Kovacevic N, Vauhan PR, "Delayed collapse of cut slopes in stiff clay, Geotechnique 1997;47(5):953–82.

[25] Stark TD, Eid HT, "Performance of three-dimensional slope stability methods inpractice, J Geotech Eng, ASCE 1998; 124:1049–60.

[26] Totonchi A, Askari F, "3D Stability Analysis of Concave Slopes in plan view using Linear Finite element and Lower Bound Method, Iranian J of science and technology, 2011

[27] Totonchi A, Askari F, "NON-DIMENSIONAL SEISMIC CHARTS OF ACTIVE AND PASSIVE FORCES IN RETAINING WALLS USING STRESS FIELDS, Journal of Applied sciences, 2011, DOI: 10.3923

[28] Ugai K. Three-dimensional stability analysis of vertical cohesive slopes. Soils Found 1985; 25(3):41–8.

[29] Viratjandr C, Michalowski RL, "Limit analysis of submerged slopes subjected to water drawdown, Can Geotech J 2006; 43:802–14.

[30] Wei WB, Cheng YM, Li L, "Three-dimensional slope failure analysis by the strength reduction and limit equilibrium methods, Comput Geotech J.

[31] Zhu DY, " A method for locating critical slip surfaces in slope stability analysis, Can Geotech J 2001; 38:328–37.

[32] Psarropoulos PN, Tsompanakis Y, " Stability of tailings dams under static and seismic loading, Can Geotech J 2008; 45:663–75.

[33] Chen J, Yin JH, Lee CF. Upper bound limit analysis of slope stability using rigidfinite elements and nonlinear programming. Can Geotech J 2003; 40:742–52.

[34] Chen Z, Wang X, Haberfield C, Yin J-H, Wang Y, " A three-dimensional slopestability analysis method using the upper bound theorem part I: theory and methods, Int J Rock Mech Min Sci 2001; 38:369–78.

[35] Taylor DW. Stability of earth slopes. J Boston Soc Civ Eng 1937;24:197–246