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The Limitations of TORA Software in Solving Linear Programming Problems: Case of the Unrevised Simplex Method

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Abstract- In this paper, in the case of the Unrevised Simplex Method (USM), we show that the TORA software is applicable only in solving Linear Programming Problem (LPP) when the matrix associated to the initial basis is an identity matrix. We show that if the matrix associated to the initial base is any, we obtain the same result as that obtained using the TORA software.

Keywords- *TORA Software, Linear Programming Problem, Unrevised Simplex Method*

I. INTRODUCTION

Operations Research (OR) is a statistical tool [1, 2] which was developed during the Second World War. Programming which started in the United Kingdom as a part of OR soon spread to the United States of America [3, 4, 5]. The USM is among the popular methods to solve the general LPP. George B. Dantzig [3] in the year 1947 formulated the general LPP and devised the USM for solving these LPP. The USM came to solve optimization problems of objective functions, moving to sides of a polygon constraint or on the edges of the polyhedron constraints [6, 7, 8]. Here we try to contrast the LLP solving using the USM and the TORA software, one of the reference softwares in LPP [9, 10].

II. MATHEMATICAL FORMULATION AND METHODOLOGY

A. Mathematical Formulation

Let:

$$\begin{cases} Max\ Z = CX \\ subject\ to \\ AX = b \\ X \geq 0 \end{cases} \tag{1}$$

be a linear program written in the standard form.

A is an m×n matrix of rank m.

Let J be any basis of A, i.e. the corresponding matrix is identity or not.

A = $[A^J,\ A^{\bar{J}}]$,

X = $\begin{bmatrix} x_J \\ x_{\bar{J}} \end{bmatrix}$,

C = $[c^J,\ c^{\bar{J}}]$,

where A^J is the matrix relating to the basic variables, $A^{\bar{J}}$ is the matrix relating to the non- basic variables, x_J are the basic variables and $x_{\bar{J}}$ the non- basic variable, c^J the cost vector related to the base J and $c^{\bar{J}}$ the cost vector not related to the base J.

In [7, 9] we set $\Pi = c^J(A^J)^{-1}$. The linear program ("Eq. 1") becomes :

$$\begin{cases} Max\ Z = \Pi\, b + \hat{C}X \\ subject\ to \\ \hat{A}\,X = \hat{b} \\ X \geq 0, \end{cases}$$

where $\hat{C} = C - \Pi A$ is the reduced cost vector ; C the cost vector ;

$\hat{A} = [I_m\ , (A^J)^{-1}A^{\bar{J}}]$;

$\hat{b} = (A^J)^{-1}b$;

I_m is an identity matrix of order m ;

$\Pi = c^J(A^J)^{-1}$ is the multiplicative vector relating to the base J or price vector.

Thereby, the linear program is written in canonical form with respect to the basis J. The J basis associated solution is:

$$\begin{cases} x_{\bar{J}} = 0 \\ x_J = (A^J)^{-1}b. \end{cases}$$

If $\hat{C} \leq 0$ then J is an optimal basis and Max Z= Π b.

In the case of the linear program:

$$\begin{cases} Min\ Z = AX \\ subject\ to \\ AX = b \\ X \geq 0, \end{cases}$$

if $\hat{C} \geq 0$ then J is an optimal basis and Min Z = Π b.

B. *Methodology*

In the case of maximization [7], if the reduced costs are negative or zero, then x_J is optimal and the USM is finished. Otherwise, the method continues.

In the case of minimisation if the reduced costs are positive or zero, then x_J is optimal and the USM is finished. Otherwise, the method continues. For both cases (maximisation and minimisation), if x_J is not optimal, we choose an index h corresponding to a non-basis variable $(x_h = 0)$ for which the reduced cost is the most positive (maximisation case) or the most negative (minimisation case). If there is ex aequo, an arbitrary choice is made. Thus, for both cases, h will be the index of the new variable. In [7], the value of x_h is

$$\min\left\{\frac{x_i}{y_{ih}}, y_{ih} > 0\right\} = \frac{x_s}{y_{sh}},$$

where:

$$y_{ih} = (A^J)^{-1} A_h,$$

i takes the values of the basis J, A_h is the h^{th} column of matrix A and the variable x_s is removed from the basis.

C. *TORA Software and the USM for LPP*

Following TORA Software conception, to solve the LPP using the USM for the next two linear programs:

$$\begin{cases} Max\ Z = CX \\ \ subject\ to \\ \quad AX = b \\ \quad\ X \geq 0 \end{cases}$$

and

$$\begin{cases} Min\ Z = AX \\ \ subject\ to \\ \quad AX = b \\ \quad X \geq 0, \end{cases}$$

the basis J is not any but has to be related to the identity matrix, obtained after adding the slack variables.

III. RESULTS

A. *Illustration Using TORA Software*

Let

$$\begin{cases} Max\ Z = 2\,x_1 + 2\,x_2 + 6x_3 \\ \qquad subject\ to \\ \quad x_1 + x_2 + 3x_3 \leq 48 \\ 4x_1 + x_2 + 6x_3 \leq 60 \\ \quad x_1, x_2, x_3 \qquad \geq 0 \end{cases} \tag{2}$$

be a linear program.

With TORA software [9], the solution is obtained after the iteration 3:

INPUT GRID - LINEAR PROGRAMMING	x1	x2	x3	Enter <, >, or =	R.H.S.
Var. Name					
Maximize	2,00	2,00	6,00		
Constr 1	1,00	1,00	3,00	<=	48,00
Constr 2	4,00	1,00	6,00	<=	60,00
Lower Bound	0,00	0,00	0,00		
Upper Bound	infinity	infinity	infinity		
Unrestr'd (y/n)?	n	n	n		

Figure 1. Input grid Linear Programming

Next Iteration | All Iterations | Write to Printer

Iteration 1						
Basic	x1	x2	x3	sx4	sx5	Solution
z (max)	-2,00	-2,00	-6,00	0,00	0,00	0,00
sx4	1,00	1,00	3,00	1,00	0,00	48,00
sx5	4,00	1,00	6,00	0,00	1,00	60,00
Lower Bound	0,00	0,00	0,00			
Upper Bound	infinity	infinity	infinity			
Unrestr'd (y/n)?	n	n	n			
Iteration 2						
Basic	x1	x2	x3	sx4	sx5	Solution
z (max)	2,00	-1,00	0,00	0,00	1,00	60,00
sx4	-1,00	0,50	0,00	1,00	-0,50	18,00
x3	0,67	0,17	1,00	0,00	0,17	10,00
Lower Bound	0,00	0,00	0,00			
Upper Bound	infinity	infinity	infinity			
Unrestr'd (y/n)?	n	n	n			

Figure 2. Iteration 1 & 2

Iteration 3						
Basic	x1	x2	x3	sx4	sx5	Solution
z (max)	0,00	0,00	0,00	2,00	0,00	96,00
x2	-2,00	1,00	0,00	2,00	-1,00	36,00
x3	1,00	0,00	1,00	-0,33	0,33	4,00
Lower Bound	0,00	0,00	0,00			
Upper Bound	infinity	infinity	infinity			
Unrestr'd (y/n)?	n	n	n			

Figure 3. Iteration3

Thus, the solution is:

Max Z = 96

$x_1 = 0,\ x_2 = 36,\ x_3 = 4.$

B. *Illustration with the USM Without Table*

The LPP ("Eq. 2") under his standard form is:

$$\begin{cases} Max\ Z = 2\,x_1 + 2\,x_2 + 6x_3 \\ \qquad subject\ to \\ \quad x_1 + x_2 + 3x_3 + x_4 = 48 \\ 4x_1 + x_2 + 6x_3 + x_5 = 60 \\ \quad x_1, x_2, x_3, x_4, x_5 \qquad \geq 0 \end{cases} \tag{3}$$

From the LPP ("Eq. 3"),

A= $\begin{pmatrix} 1 & 1 & 3 & 1 & 0 \\ 4 & 1 & 6 & 0 & 1 \end{pmatrix}$, C = (2, 2, 6, 0, 0), b= $\begin{pmatrix} 48 \\ 60 \end{pmatrix}$.

Taking for example, J = {2, 3}, the corresponding matrix is:

$$A^J = \begin{pmatrix} 1 & 3 \\ 1 & 6 \end{pmatrix},\ (A^J)^{-1} = \begin{pmatrix} 2 & -1 \\ -\frac{1}{3} & \frac{1}{3} \end{pmatrix},$$

$$x_J = (A^J)^{-1} b = \begin{pmatrix} 36 \\ 4 \end{pmatrix} = \begin{pmatrix} x_2 \\ x_3 \end{pmatrix}.$$

The associated basic solution is:

$$X=\begin{pmatrix}0\\36\\4\\0\\0\end{pmatrix}. \quad \Pi = (2, 6)\begin{pmatrix}2 & -1\\ -\frac{1}{3} & \frac{1}{3}\end{pmatrix} = (2, 0).$$

$$\hat{C} = (2, 2, 6, 0, 0) - (2, 0)\begin{pmatrix}1 & 1 & 3 & 1 & 0\\ 4 & 1 & 6 & 0 & 1\end{pmatrix} = (0, 0, 0, -2, 0).$$

As $\hat{C} \leq 0$ then J is an optimal basis and the solution is:

$$\text{Max } Z = \Pi\, b = (2, 0)\begin{pmatrix}48\\60\end{pmatrix} = 96$$

$x_1 = 0,\ x_2 = 36,\ x_3 = 4.$

IV. CONCLUSION AND FUTURE WORK

In this paper we have showed that the TORA software which is one of the reference softwares in Linear Programming, is incomplete. We can't use the TORA software in solving Linear Programming, Problem (LPP) when the matrix associated to the initial basis is not an identity matrix, even if we have the same results.

In future we hope to get new TORA software in solving Linear Programming Problem (LPP) when the matrix associated to the initial basis is any.

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