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# FDTD Method Simulation with Perfect Matching Layer at GSM Frequency

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***Abstract***-The finite-difference time-domain (FDTD) method is one of the most widely used computational methods in electromagnetic. This paper study the simulation of wave propagation using one and two Finite Difference Time Domain method has been studied. It being used to study the time evolution behavior of electromagnetic field by solving the Maxwell's equation in time domain using perfect matching layer boundary condition. The basic idea is this that the amount of reflected waves is dictated by the intrinsic impedance of the two medium. The software is developed using Matlab programming language. Numerical examples validate the software.

***Keywords-*** *Finite Difference Time Domain (FDTD) Method, Perfectly Matched Layer (PML), Simulation GSM Frequency*

## I. INTRODUCTION

Absorbing boundaries for wave given a solution on a discrete grid, a boundary condition is a rule to set the value at the edge of the grid. For example, a simple Dirichlet boundary condition sets the solution to zero at the edge of the grid, which will reflect waves that hit the edge [1]. The finite difference time domain method (FDTD) gives pleasant results if the discretization is fine enough. In the FDTD method for electromagnetic wave propagation problems in unbounded media, artificial boundary conditions have to be used to eliminate the reflections from the edge of the finite computational domain. These boundary conditions are known as absorbing boundary conditions, because they are developed to absorb outgoing waves. Among many different absorbing boundary conditions, the perfectly matched layers (PML). Recently, provide highly effective absorption to the outgoing waves. Because the PML is a material absorbing boundary condition, it is ideal for parallel computation, because unlike most other absorbing boundary conditions, only one set of code is required for the computational domain and for its boundary [2]. The PML was first introduced for the Maxwell's equations many research [3-7]. There are two standard methods [8, 9] that have been used to study the perfect matching property of the PML. The approach [9] uses plane wave analysis and only accounts for propagating modes. Here, we use the technique [8] which is rooted in the construction of general solution to the wave equation in Laplace-Fourier space. The technique [8] is more general since it includes both the propagating mode regime and the evanescent mode regime. Robert et al studied an adaptive finite element method for the wave scattering with transparent boundary [10]. Also viewpoint allows PMLs to be derived for inhomogeneous media such as waveguides, as well as for other coordinate systems and wave equations [11]. In this work we studied the simulation of global system mobile wave propagation using one and two Finite Difference Time Domain method. It being used to study the time evolution behavior of electromagnetic field by solving the Maxwell's equation in time domain using perfect matching layer boundary condition.

## II. THEORY AND MODEL

The efficient of absorption boundary condition the perfect matched layer has been developed. The basic idea is that if the wave is propagating in medium and it sticks another medium, the amount of reflection is depend on the impedance of two medium. The impedance determine by the permittivity and permeability of the medium as

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}$$

We assume that the permeability is constant. However if μ is change with ε, then η remained constant. The normalized Maxwell's equations are:

$$\frac{\partial \tilde{D}}{\partial t} = \frac{1}{\sqrt{\varepsilon_o \mu_o}} \nabla x H \quad (1)$$

$$\frac{\partial H}{\partial t} = -\frac{1}{\sqrt{\varepsilon_o \mu_o}} \nabla x \tilde{E} \quad (2)$$

Equation (1) and (2) reduced to:

$$\frac{\partial \tilde{D}_z}{\partial t} = \frac{1}{\sqrt{\varepsilon_o \mu_o}} \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) \quad (3)$$

$$\frac{\partial H_x}{\partial t} = -\frac{1}{\sqrt{\varepsilon_o \mu_o}} \frac{\partial E_z}{\partial y} \quad (4)$$

$$\frac{\partial H_y}{\partial t} = -\frac{1}{\sqrt{\varepsilon_o \mu_o}} \frac{\partial E_z}{\partial x} \tag{5}$$

We are going to the Fourier domain in time, so $\partial / \partial t$ becomes ($j\,\omega$),the equations above becomes:

$$j\omega \tilde{D}_z = c_o.(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}) \tag{6a}$$

$$j\omega H_x = -c_o \frac{\partial E_z}{\partial y} \tag{6b}$$

$$j\omega H_y = c_o \frac{\partial E_z}{\partial x} \tag{6c}$$

To form PML that the impedance going from the background medium to the perfect matched layer, it must be constant as:

$\eta_o = \eta_m = \sqrt{\frac{\mu^*_{Fx}}{\varepsilon^*_{Fx}}} = 1$, where $\varepsilon^*_{Fx} = 1/\varepsilon^*_{Fy}$ and $\mu^*_{Fx} = 1/\mu^*_{Fy}$

Using the relation $\varepsilon^*_{Fm} = \varepsilon_{Fm} + \sigma D_m / j\omega\varepsilon_o$ and $\mu^*_{Fm} = \mu_{Fm} + \sigma D_m / j\omega\varepsilon_o$, that is $\mu^*_{Fx} = \varepsilon^*_{Fx} = 1$.

Then,

$$\eta_o = \eta_m = \sqrt{\frac{\mu^*_{Fx}}{\varepsilon^*_{Fx}}} = \sqrt{\frac{1+\sigma(x)/j\omega\varepsilon_o}{1+\sigma(x)/j\omega\varepsilon_o}} = 1$$

Starting by implementing a PML in X direction, we have:

$$j\omega \tilde{D}_z.\varepsilon^*_{Fx}(x) = c_o.(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}) \tag{7a}$$

$$j\omega H_x.\mu^*_{Fx}(x) = -c_o \frac{\partial E_z}{\partial y} \tag{7b}$$

$$j\omega H_y.\mu^*_{Fy}(x) = c_o \frac{\partial E_z}{\partial x} \tag{7c}$$

The value of $\mu^*_{Fx} = \varepsilon^*_{Fx} = 1$ leads to the new formula as:

$$j\omega(1+\frac{\sigma D(x)}{j\omega\varepsilon_o})D_z = c_o.(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}) \tag{8a}$$

$$j\omega(1+\frac{\sigma D(x)}{j\omega\varepsilon_o})^{-1} H_x = -c_o \frac{\partial E_z}{\partial y} \tag{8b}$$

$$j\omega(1+\frac{\sigma D(x)}{j\omega\varepsilon_o})\ H_y = -c_o \frac{\partial E_z}{\partial x} \tag{8c}$$

Now, put equations (8) into the FDTD formula to become as:

$$D_z^{n+1/2}(i.j) = \left\{\frac{1-\sigma_D(i).\Delta t/(2.\varepsilon_o}{1+\sigma_D(i).\Delta t/(2.\varepsilon_o}\right\}.D_z^{n-1/2}(i.j) + \tag{9}$$
$$+\left\{\frac{1}{1+\sigma_D(i).\Delta t/(2.\varepsilon_o}\right\}.0.5.\begin{bmatrix} H_y^n(i+1/2,j) - H_y^n(i-1/2,j) \\ -H_x^n(i,j+1/2 - H_y^n(i,j-1/2) \end{bmatrix}$$

where $\frac{\Delta t}{\Delta x} c_o = \frac{\Delta x/(2c_o)}{\Delta x} c_o = \frac{1}{2}$

and for magnetic field equation we have:

$$H_y^{n+1/2}(i+1/2,j) = \tag{10}$$
$$\left\{\frac{1}{1+\sigma_D(i+1/2).\Delta t/(2\varepsilon_o)}\right\} H_y^n(i+1/2,j) +$$
$$+\left\{\frac{1-\sigma_D(i+1/2).\Delta t/(2\varepsilon_o)}{1+\sigma_D(i+1/2).\Delta t/(2\varepsilon_o)}\right\}.0.5[E_z^{n+1/2}(i+1,j) - E_z^{n+1/2}(i,j)]$$

## III. RESULTS AND DISCUSSIONS

The PML boundaries will probably work more accurately if placed a little farther away from the scattered. For fine discretization, four cells may be in the near field and the PML will absorb reactive fields which may consist of important information such as in the case of input impedance. The parameters are calculated at i+1/2 because of the position of $H_y$ in the FDTD grid, and the PML is implemented in Matlab program. Figure (1) simulation of 2D electric field(Ex) FDTD with perfect matching layer at frequency 900MHz and 100 time steps, the effectiveness of an 8point PML. With the source offset five cells from center in both X and Y direction. In Figure (2) the simulation of 2D Electric field (Ez) by FDTD method with perfect matching layer at frequency 900MHz and 100 time steps is presented. Another simulation shown in Figure (4), which simulation of 1D Electric field (Ez) by FDTD method with perfect matching layer at frequency 900MHz and 35 time steps.

## IV. CONCLUSION

PML is widely used and has become the absorbing boundary technique of choice in much of computational electromagnetism. The basic idea is this work that the amounts of reflected waves are dictated by the impedance of the two medium. The software is developed using Matlab programming language. Numerical examples validate the software:

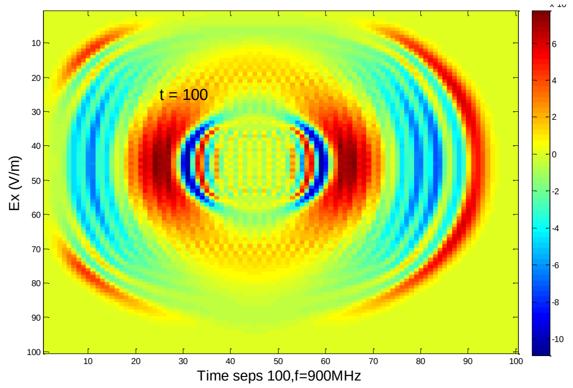


Figure 1. Simulation of 2D electric field (Ex) FDTD with perfect matching layer at frequency 900MHz and 100 time steps.

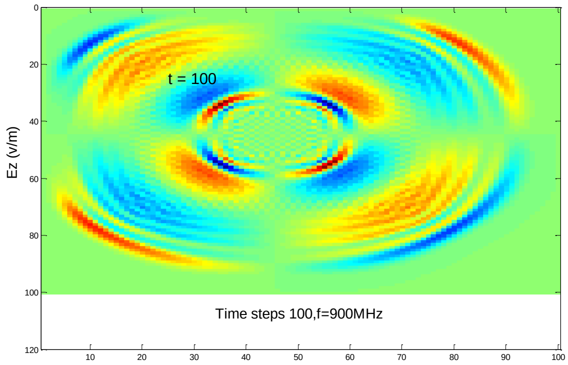


Figure 2. Simulation of 2D Electric field (Ez) by FDTD method with perfect matching layer at frequency 900MHz and 100 time steps.

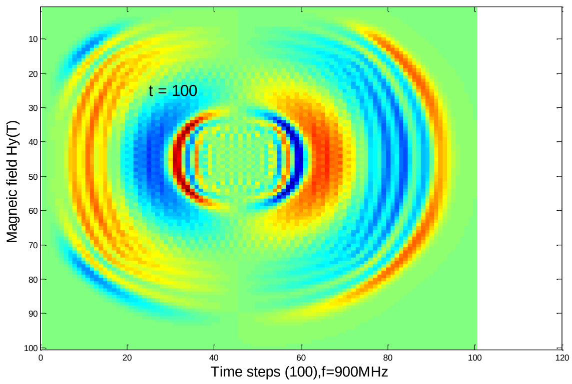


Figure 3. Simulation of 2D Magnetic field (Hy) by FDTD method with perfect matching layer at frequency 900MHz and 100 time steps

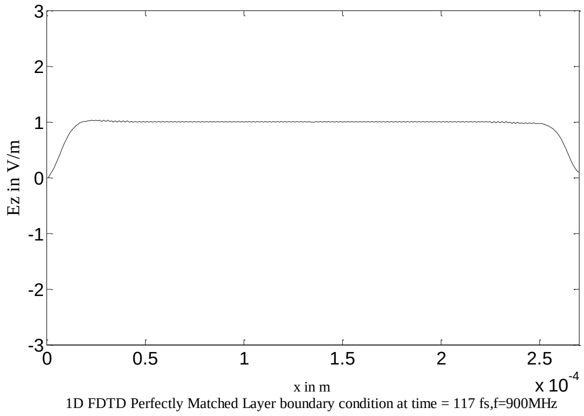


Figure 4. Simulation of 1D Electric field (Ez) by FDTD method with perfect matching layer at frequency 900MHz and 35 time steps

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