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Types of Warrant in Mathematical Argumentations of Prospective-Teacher

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Abstract- This paper discusses types of *warrant* in mathematical argumentations stated by prospective-teacher. To solve mathematics problems, a problem solver needs argumentations to determine, yield, and bolster reasonable solution. Mathematical argumentations stated by students can be analyzed using Toulmin scheme that consists of data, claim, warrant, backing, rebuttal, and qualifier. This study focused on warrant because warrant is one of determinants of the quality of an argumentation. This study aims to describe types of warrant in mathematical argumentations. This study applied qualitative approach by collecting some data from written result, think aloud and interview. The subjects of this study are asked to investigate the truth of mathematical statements. Researchers choose prospective-teacher of Mathematics Education Study as subject, because they will be teachers of mathematics, who will give influences in the development of students' thinking process in the mathematical argumentation. The result shows that there are three types of warrant in mathematical argumentation stated by the students, they are structural-intuitive, inductive and deductive. Both inductive and structural-intuitive warrants are considered as non-deductive. Non-deductive warrant-type is used to reduce uncertainty of the conclusion. Besides, the subjects used deductive warrant-type to remove uncertainty of the conclusion.

Keywords- *Warrant, Mathematical Argumentation, Prospective Teacher*

I. INTRODUCTION

Math is a question that requires an answer, but the answer cannot be known immediately [1]. A math problem requires an ability to define, generate and support reasonable actions to get the right answer. Problem solving requires arguments to justify the solutions and actions [2] [3] [4] [5] [6]. Thus, developing the ability of argumentation is needed by students, so that they can provide a brief description or explanation to support or refuse an opinion or idea.

Argument is an ability to link data to make a claim [7]. Argument is an important type of informal reasoning. It is the center of intellectual abilities involved in solving problems, making judgments and decisions, and formulating ideas and beliefs [5]. By having the argumentation ability, students can leave their indecision and doubt in solving a problem, they also get more freedom in choosing an idea, they can even propose a rational response in resolving the matter.

Even though the ability of argumentation is needed to solve the problem, many university students and high school students failed to state an argument. Students are not proficient to build a convincing argument [2]. Argumentative reasoning skills have not been practiced evenly across all school environments, and the ability to make a reasoned assessment should be part of the ability to "think well" [8]. The difficulties and weaknesses of the adolescents and young adults subject in constructing and developing arguments [9] [10]. The students with various different ages and he found that only a few of them are consistently able to develop a quality argument [2]. Thus, research related to mathematical argumentation is important to be investigated further as an evaluation of students' thinking process and the possibility of emerging models of mathematical arguments.

Arguments can occur in a dialog or non-dialog [11]. Examples of the arguments that occur in a dialog could be found during the critical discussion, in which each participant tries to show a correct ways in addressing the arguments to other participants. Examples of non-dialogue argument are planning or problem solving. Non dialog argument is interesting to study because the planning activities or the completion of an interactive reasoning is conducted by a subject himself, in which the same person alternately plays the role of initiator and responder. Approaches and debates are conducted by the subject toward himself.

An argument needs to be analyzed using a richer format so that people do not only distinguish between premise and conclusion [12]. Therefore, Toulmin proposed a layout which is known as Toulmin Scheme. The Toulmin scheme consists of the data (D), claim (C), warrants (W), backing (B), rebuttal (R) and qualifier (Q). Data are the facts used to support the claim. Claim is a proposition that is supported by the data. Warrant is a guarantee for data in supporting the claim. Warrant is supported by a backing, while backing presents further evidence which is the legal basis as the foundation of warrant. Rebuttal is an exception condition for arguments, and qualifier can reveal the power level of data provided by the warrant to the claim.

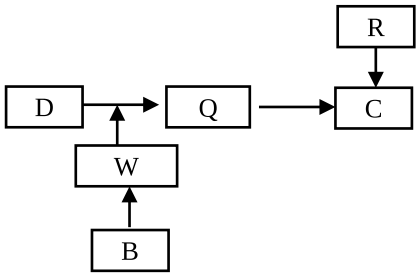

Figure 1. Toulmin Scheme on general argumentation

This paper focuses on the warrant component used in mathematical argumentation, the significance of research in revealing and understanding the mathematical argument particularly on warrant. It is an opportunity to conduct research with how someone builds a warrant in math by trying to show the Orefield-dependence variability [12]. The warrant of an argument could be subject to certain restrictions, where these limits must be observed so that the truth value of an argument is not disputed. There are three important parts which are called the core of the argument, they are: data, conclusions, and warrants [13]. When someone presents an argument, he/she is trying to convince the audience on particular statement referred as a conclusion. To support the conclusion, the presenter usually shows evidence or data. Presenter's explanation on why the data support the conclusion is called warrant. At this stage, audiences can receive the data but they may reject the explanation that the data is setting the conclusion. In other words, the authority of the warrant may be challenged. If this happens, the presenter is required to provide additional supports to justify the warrant, and therefore the core of the argument is valid.

Types of warrants as follows: inductive, structural-intuitive and deductive [14]. Inductive Warrant is a foundation obtained from a process involving the evaluation of one or more specific cases. Structural-intuitive warrant is a foundation obtained from intuition (intuitive thinking) about the structure of a person's internal representation. Deductive warrant is a foundation obtained from formal mathematical justification process used to ensure the general conclusions. [14] focused more on the qualifier model of each kind of warrant. Even though, they have not been described warrants in a systematic and structured form. Therefore, this paper will describe the mathematical argument based on the type of warrant.

Researchers choose students of Mathematics Education program as research subject, because they are prospective teachers of mathematics in the future, which will be influential in the development of students' thinking process in mathematical arguments. Students should be able to make an argument to establish the validity of the allegation [15]. The argument used by the students depends on the formation of theorem culture in the classroom, the nature of the task, and certain types of reasoning which is emphasized by the teacher [16]. So that the actions of the teachers can encourage students to define, write and justify a class discussion.

Based on the outlined theory and previous research related to mathematical argument, Researchers intend to describe the type of warrant in mathematical argument generated by prospective teachers in solving mathematical problems. The type of warrant refers to the theory of [14].

II. METHOD

Researchers use qualitative approach, because it is the most relevant approach to help them achieving the objectives of this research. This study aims to determine and describe the mathematical argumentation models of students of Mathematics Education Program in solving problems. The subjects are the 6th (sixth) semester students because they have been already studying the concept of relations.

There are two kinds of instruments used in this research; they are the main instruments and auxiliary instruments. The main instrument is the researchers themselves, while the auxiliary instruments consist of two types; they are mathematics problem and interview guidelines. Math is used to describe a type of warrant in mathematical argument. Once completed, students were asked to express verbally what he was thinking as much as possible during the process of completing. Researchers used the video recording to record the activities of the subject during the process of solving problems. The problem is as follows:

Investigate The Truth Of The Following Mathematical Statement:

If $\mathbb{Z}$ is a set of integer number and P is a binary relation on $\mathbb{Z}$ which is defined as $P = \{(a, b) \in \mathbb{Z}\times\mathbb{Z} \mid a - b = 7k$ *, for a whole number of k*$\}$, so the binary relation of P on the set of $\mathbb{Z}$ is antisymmetric

Further interviews were conducted based on the subject's written answers. Researchers used unstructured interviews to uncover deeper thinking process which is done by the students during the process of solving problems and not revealed during the "think aloud" process. Broadly speaking, the interview is conducted to find out what the subject thinks while concluding something and taking a step. Questions might be "How do you think of this?" Or "what's on your mind today?". Researchers used video recording to record the activity during the interview process.

The data obtained from the process of interview and "think aloud" are transcripted and analyzed. Researchers analyze the data using three stages of qualitative data analysis activities by [17] and six stages of the analysis and interpretation of qualitative data by [18]. The stages of data analysis in this study are: (1) transcripting the data; (2) reducing the data; (3) encoding the data; (4) checking the validity of the data (data triangulation); (5) reviewing the data; (6) interpreting the findings; (7) drawing the conclusions.

III. RESULTS AND DISCUSSION

Students of mathematics education in STKIP PGRI Jombang are involved in this research. It was conducted on February 25 to March 22, 2016. The arguments of students were analyzed using Toulmin scheme. Certain parts of the Toulmin scheme were not explicitly verbalized by the subject. As like previous researchers namely [1] [14], this research also faced the same problem as follows: the parts of Toulmin scheme were not explicitly verbalized by the subject. Therefore, the data does not contain written reply only, but also explanation of behavior and words spoken by the subject, even though the subject did not directly state it. The warrant resulting in the mathematical argumentation of the subjects could be described as follows:

A. The Inductive Warrant-Type

Inductive Warrant occurs when the subjects ensure themselves and persuade others on the truth of allegations by evaluating the allegation in one or more specific case to reduce the uncertainty of a conclusion [14]. Inductive warrant produces a conclusion derived from the specific cases into common traits, based on evaluating on these special events. The process to formulate an inductive warrant conclusion can be described in the following scheme:

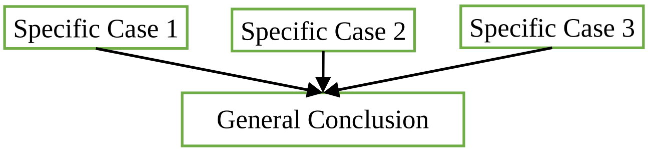

Figure 2. Conclusion with Inductive Warrant

The inductive warrant-type in mathematical argumentation of the students is shown as follows: Starting at the subject named RK (initial name) who stated ideas namely ℤ, binary relation P, ℤ×ℤ, and numbers count, RK as the subject mentioned any used ideas in details, namely ℤ which is a set of integers. The subject then revealed the elements of ℤ which is {. , ., -3, -2, -1, 0, 1, 2, 3,. , }, Because many elements of the set of integers was infinite, subject only wrote part of elements of the set of integers which is -3, -2, -1, 0, 1, 2, 3, and for the other elements, subject just provided symbol ". , . ". Furthermore, the subject described the binary relation of P. The binary relation of P is a subset of the ℤ×ℤ, ℤ×ℤ = {. . ., (-3, -3), (-3, -2), (-3, -1), . . .}. Subject stated that many elements of ℤ are infinite, then he wrote the elements of ℤ × ℤ elements as (-3, -3), (-3, -2), (-3, -1), and for the other elements the subject just provides symbol ". . . ". The followings are the results of the written data of the subject (RK).

$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
P is a binary relation on ℤ.
By definition:
P is called the binary relation in the set ℤ, if P is a non-empty set of ℤ × ℤ.
ℤ × ℤ = {. . ., (-3,-3), (-3,-2), (-3,-1), …}

The Subject (RK) detailed P = {(a, b) ∈ℤ×ℤ | a – b= 7k, for a whole number of k}. Furthermore, the subject of RK was looking for elements of P by operating several elements of ℤ that meets a – b = 7k, for k is a whole number. Subject RK take the first example -7, -7 substituted to a – b = 7k, -7 - (-7) = 0 so that k = 0, because 0 is a valid whole number then (-7, -7) element of P. Subject RK took second example which is 7 and -7, 7 and -7 substituted to a – b = 7k, 7 - (-7) = 14, so that k = 2, because 2 is a count of the number (7, -7) elements of P. Furthermore the subject RK took the third example which is 14, 14 is substituted to a – b = 7k, 14 – 14 = 0 so that k = 0, because 0 is a whole number then (14, 14) elements of the subject P. RK took fourth example is -14, -14 is substituted into a - b = 7k, -14 - (-14) = 0 so that k = 0, because 0 is a valid whole number then (-14, -14) is element of P. Thus, elements of P are (-7, -7), (7, -7), (14, 14), (-14, -14). For the other elements of P, the subject wrote "...".

$P \subseteq \mathbb{Z} \times \mathbb{Z}$
P = {(a, b) ∈ℤ×ℤ | a – b= 7k, k is whole number}
P = {(-7, -7), (7, -7), (14, 14), (-14, -14), . . .}
How to determine the members of P:

- Take (-7, -7)
  (-7) – (-7) = 7k
  (-7) + 7 = 0
  (-7) + 7 = 7 (0), 0 ∈ whole number
- Take (7, -7)
  7 – (-7) = 7k
  7 + 7 = 14
  7 + 7 = 7 (2), 2 ∈ whole number
- Take (14, 14)
  14 – 14 = 7k
  14 – 14 = 0
  14 – 14 = 7 (0), 0 ∈ whole number
- Take (-14, -14)
  (-14) – (-14) = 7k
  (-14) + 14 = 0
  (-14) + 14 = 7 (0), 0 ∈ whole number

Furthermore, the subject of RK checked whether P = {(-7, -7), (7, -7), (14, 14), (-14, -14), . . .) is antisymmetric or not. Firstly, the subject wrote the definition of antisymmetric that if aRb, bRa so a = b, ∀a, b∈ℤ. Subject later investigatied two elements, namely P (-7, -7) and (14, 14). The following data is subject's written data:

Whether the relation P on the set ℤ is antisymmetric?
The antisymmetric definition is that if aRb, bRa then $a = b$, $\forall\, a, b \in Z$
Example:

- $a = -7, b = -7$
  $-7R-7, -7R-7$ so -7 = -7 and (-7, -7) ∈ ℤ
- $a = 14, b = 14$
  $14R14, 14R14$ so 14 = 14 and (14, 14) ∈ ℤ

Based on the above definition and explanation can be concluded that the binary relation P on the set Z is antisymmetric is true statement

Subject RK concluded that the statement “If ℤ is the set of integers and P is a binary relation on ℤ defined as P = {(*a*, *b*) ∈ℤ×ℤ | *a* – *b* = 7*k*, for a whole number of *k*}, then the binary relation of P on the set of ℤ is antisymmetric” is a true statement on the basis of two investigated.examples. Modal qualifier by the subject is called "probable" based on the theory of Inglis, Ramos and Simpson (2007). The following data is subject’s “think aloud”:

Subject :". , , Ohhh it's partly a bit of members of P, in fact there are very many members of P. From these examples it turns out they all meet antisymmetric nature, if *a* is related to *b* and *b* is related to *a*, then *a* = *b*, and *a*, *b* are the elements of ℤ. Based on the definition and completion above, it is true that the binary relation of P on the set ℤ is antisymmetric ".

Subject tried to assure himself by evaluating two specific incidents of element P which is (-7, -7) and (14, 14) whether it meets antisymmetric nature or not. This such action referred to as "naive empiricism" according to Balacheff (1988). The subject confirms the truth of the results after verifying some cases. The model of subject’s mathematical argument using Toulmin scheme appears in Figure 3.

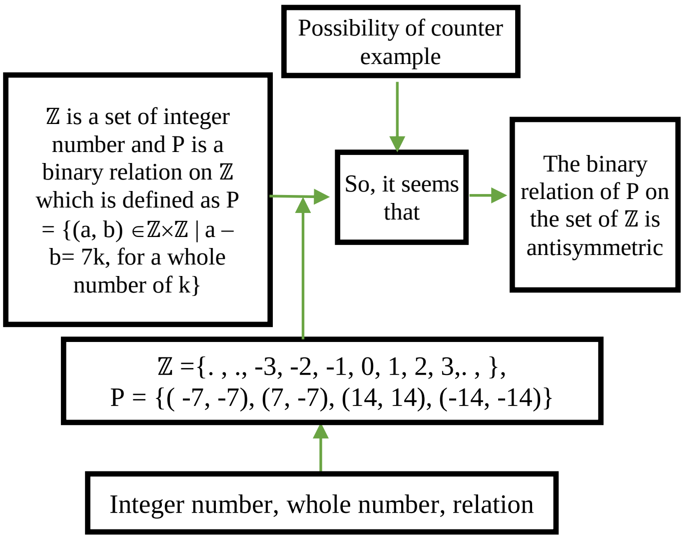

Figure 3. Subjek (RK) argument

B. *The Intuitive Warrant-Type*

Intuitive Warrant occurs when the subject uses observation, and also some types of mental structures, could be visual or not, to persuade them to the conclusion [14]. The role of intuition in mathematics has been recorded by many great mathematicians, for example [19], [20] and [21] and philosopher of mathematics, for example [22]. But the first major study of intuition in mathematics education conducted by Fischein in 1987. Intuition is often described as a kind of spontaneous thinking received directly from individual confidence when information is associated with past experience, it may not be identical to the beliefs of others, but can understood by others [23]. So, intuitive warrant in this paper is when the subject receives directly the fact as a provision which is considered to be true or proven by itself and global nature.

Intuitive warrant in student’s mathematical argument started after reading questions, the subject named SD (initials) revealed the ideas used, they are: ℤ integers, whole numbers, and binary relation of P. The detail is as follows: ℤ is an integers starting from ∞ to ∞, the set of whole numbers is 0, 1, 2- , , .∞, and the binary relation of P is described as{(*a*, *b*) ∈ℤ×ℤ | *a* – *b* = 7*k*, *k*∈ whole number. Once the (subject) SD detailed the ideas immediately and confidently declaring that the statement is true, the subject justifies his argument by intuition that an integer does not apply commutative rules on subtraction. The following is the written data of the subject:

If ℤ is an integers = $\{-\infty, \ldots, \infty\}, k = \{0, 1, 2, \ldots, \infty\}$
P = {(*a*, *b*) ∈ℤ×ℤ | *a* – *b* = 7*k*, *k* ∈ whole number}

Subject SD tried to show a counterexample by subtracting whole numbers of 2 integers when *a* is an integer that is less than 0 and *b* is a bigger integer or equal to 0 then *a* – *b* = 7*k*,, where 7k is a negative number, so 7k is not whole numbers, However, when the position of *a* and *b* is reversed i.e. *b* – *a* = 7*k*, where 7k is a valid whole number, subject gives an example in the condition of *a* = 2 and *b* = -5. The followings is “think aloud” data of the subject:

Subject: "If *a*, element of integer is less than 0 and *b* integer is greater than or equal to 0 and a - b is a multiple of 7 is negative, so the relation of P does not happen because *k* is not an element of a valid whole number. Suppose a = 2 and b = -5, (2, -5), -5 - 2 = -7 and 2 - (-5) = 7, 7: 7 = 1, k = 1, 1 it is cacah. When -5 - 2 = -7 emm not whole number right?"

Researcher : "why did you make this example?"

Subject: "To show these two integers with the relation of P do not meet the criteria of antisymmetric, integer does not apply commutative rules on subtraction”

The mathematical argument of the subject (SD) is interesting because after declaring the statement "If ℤ is a set of integers and P is a binary relation on ℤ defined as *P = {(a, b) ∈ℤ×ℤ | a – b = 7k* , for k is whole number}, then the binary relation P on a set ℤ is antisymmetric" as a false statement and trying to find a counterexample, yet this subject gave another argument which is contradictive as follows: “there are some conditions of k = 0 which cause *a* – *b* = *b* – *a*”. Thus, the statement "If ℤ is a set of integers and P is a binary relation on ℤ defined as *P = {(a, b) ∈ℤ×ℤ | a – b = 7k* , for k is whole number}, then the binary relation P on a set ℤ is antisymmetric" is valid because *a* – *b* = *b* – *a* = 0, then *a* = *b*. So the binary relation of P on a set of ℤ is antisymmetric. The following is “think aloud” data of the subject:

Subject: “Unless 7k was equal to 0 with k = 0, 0 is a valid whole number. When k = 0 then *a* – *b* = *b* – *a* = 0, is consequently *a* = *b* so that the binary relation of P is antisymmetric"

It is obvious that the mathematical argument through intuitive structural warrant by the subject SD provides two different conclusions. This matches the result of the research by [14] that sometimes intuition provides support to wrong conclusions. Sometimes intuition give different conclusions by deductive reasoning that stimulate interaction or conflict [1]. Although intuition sometimes is misleading, but it is important to provide direction for mathematical research [22].

Data from these studies show that subject SD use intuitive structure to build confidence and reduce uncertainty of conclusions. Thus, the modal qualifier of the subject is called "probable" according to [14]. The model of mathematical argument of the subject through Toulmin scheme appears in Figure 4.

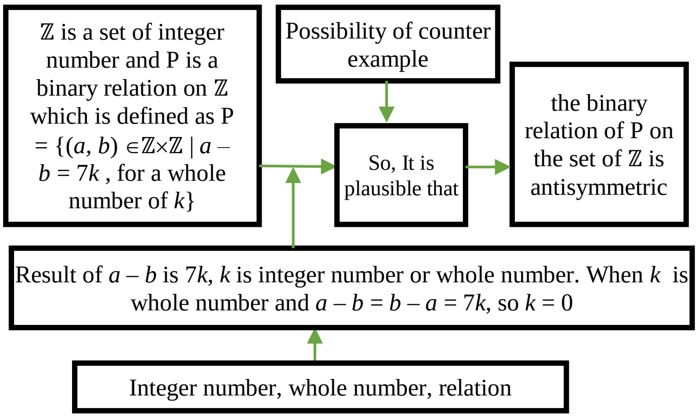

Figure 4. Subjek (SD) argument

C. The deductive Warrant-Type

The deductive warrant-type is a formal mathematical justification used to ensure the conclusion [14]. The justification can be of various kinds such as deductions from axioms, algebraic manipulation, or counter examples. The mathematical argument of the students using deductive warrant-type begins from the subject named IM (name initials) who revealed the following ideas: ℤ, binary relation P, antisymmetric. Subject IM then itemize every idea used such as antisymmetric, binary relation P and count numbers. Furthermore the subject detailed every idea expressed as follows: the definition antisymmetric defined as *a*R*b* and *b*R*a* so $a = b$ for $a, b \in$ S. The relation of P defined as $\{(a, b) \in \mathbb{Z}\times\mathbb{Z} \mid a - b = 7k$, for a whole number of $k\}$, and the whole number are 0, 1, 2, 3,. , For other valid whole numbers, the subject IM provides symbol ". , . ". The following data are the written data and "think aloud" of the subject:

Subject IM: "It's by definition 4, antisymmetric is *a* related to *b* and *b* related to *a*, for *a*, *b* element of S. $P = \{(a, b) \in \mathbb{Z}\times\mathbb{Z}$ | *a – b= 7k, for k is a whole number.* Whole numbers begins at 0, 1, 2, 3 and so on."

By definition 4
Antisymmetric is *a*R*b* and *b*R*a* so $a = b$, $\forall a, b \in$ S.
P = {(a, b) ∈ℤ×ℤ | a – b= 7k, for k is a whole number.
Whole numberis 0, 1, 2, 3, …

Subject then detailed the interpretation *a*R*b* as $a - b = 7k$ and called it Equation 1. From equation 1 it's obtained $a = 7k + b$. Subject also detailed the interpretation of *b*R*a* as $b - a = 7l$ and called it equation 2. Subject then substituted equation 1 to equation 2 to obtain $k = l$. Subject later investigated a valid whole number that satisfies $k = l$. Numbers count that satisfies $k = l$ is 0, because -0 = 0 and count any number other than 0 no meet -k = *l*, for example 1, 1 is not equal to -1, so that one does not meet $k = l$, The followings is the "think aloud" data of the subject:

Subject IM: ""Now we're making examples for *a R b*, it is *a – b= 7k*, and for *bR a* it is *b – a = 7l*. It means equation 1, equation 2. In equation 1 $a = 7k + b$ is then substituted into equation 2. Equation 2 is $b - a = 7L$, $b - (7k + b) = 7l$, here $b - 7k - 7L$ and $b - b$ so $-7k = 7L$. Here we got $-7k = 7l$ so $-k = l$.. See $-k = l$, and caca count starts from 0, 1, 2, 3 and so on. If for example cacah 1, it means $-1 = 1$, it does not fit, since -1 is not equal to 1. This is to complete $-k = l$ if we suppose the whole number is 0, it must be 0 in order to fulfill this. 0 is the numbered count. The result of $-0 = 0$ is indeed 0!"

Furthermore, the subject substituted $-k = l = 0$ to $a - b = 7k$ and $b - a = 7l$ and the result is $a = b$. The subject then concludes binary relation of P that satisfies antisymmetric rules. The followings are the written data of the subject:

-0 = 0
0
a = *b*
So P is antisymmetric because *a*R*b* and *b*R*a* → *a* = *b*

Subject concluded that the statement "If ℤ is a set of integers and P is a binary relation on ℤ defined as $P = \{(a, b) \in \mathbb{Z}\times\mathbb{Z} \mid a - b = 7k$, for k is a valid whole number} then the binary relation of P on a set of ℤ is antisymmetric " is a true statement based on the definition of antisymmetric and algebraic manipulation. Mathematical argument stated by the subject using deductive warrants has no objections and absolute truth. Modal qualifier of the subject IM in called "Certain" based on [14]. The model of subject's mathematical argument using Toulmin scheme appears in Figure 5.

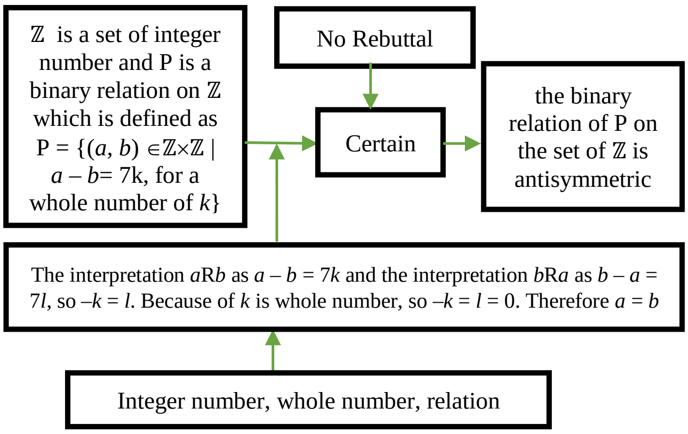

Figure 5. Subjek (IM) argument

The inductive warrant-type and structural inductive warrant-type are considered as non-deductive. There are many experts (e.g., [24] [25]) stated that non-deductive warrants are improperly used at the university student level. That is because non-deductive warrant gives no definite conclusions, while formal mathematics needs deductive warrants. However, the non-deductive warrant-type play important role in mathematical argumentations, as long as it matches suitable qualifier [14]. The data of this study supports the idea of [14]. Subjects use non-deductive warrant in mathematical argumentation to build confidence in the conclusions. Non-deductive warrant-type is used to reduce the uncertainty of the conclusions expressed by the subject. Besides, the subjects used deductive warrant-type to remove uncertainty of the conclusion.

Qualifier of the conclusion obtained from deductive warrant is certain [14]. Deductive mathematical argument to warrant a conclusion that mutlat produce and there is no objection used to doubt or reject these conclusions. However, the resulting deductive warrant subject does not give the certain qualifier [14]. While the data of this study indicate that the qualifier that was found on the deductive warrant is certain. Subjects using deductive warrant aims to eliminate the uncertainty of the conclusions expressed.

IV. Conclusion

The result shows that there are three types of warrant in mathematical argumentation stated by the students, they are structural-intuitive, inductive and deductive. Both inductive and structural-intuitive warrants are considered as non-deductive. Non-deductive warrant-type is used to reduce uncertainty of the conclusion. Besides, the subjects used deductive warrant-type to remove uncertainty of the conclusion. Because, qualifier of the conclusion obtained from deductive warrant is probable. Whereas, qualifier of the conclusion obtained from deductive warrant is certain.

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