International Journal of
Science and Engineering Investigations
vol. 7, issue 73, February 2018
ISSN: 2251-8843

Shape Optimization of Gravity Dams Using Genetic Algorithm

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Abstract-Design of massive structures such as dams often includes selecting different geometric, hydraulic, and structural parameters to meet the structural design and performance requirements and many constraints must be simultaneously satisfied to obtain an optimal solution. The trial-and-error methods can be used to find these parameters for simple structures however for massive structures it may become overwhelmingly time intensive with no guarantee that an optimal solution is achieved. Previous studies have shown that the evolutionary optimization algorithms such as genetic algorithm may be an appropriate tool for designing optimized solutions for many design cases such as topology optimization and design of more efficient structures. These successful applications can be adopted for shape optimization of concrete gravity dams. This work investigates the use of the genetic algorithm in shape optimization of gravity dams under static and dynamic loads with a variety of shape parameters. In this research, four existing dam designs are used as the case studies. Shape parameters are optimized and then were compared with the existing structures. A significant level of optimization was obtained using a genetic algorithm. The results show promising performance not only on shape optimization but also on optimizing the design parameters.

Keywords– *Evolutionary Optimization, Genetic Algorithm, Gravity Dam, Objective Function, Optimization*

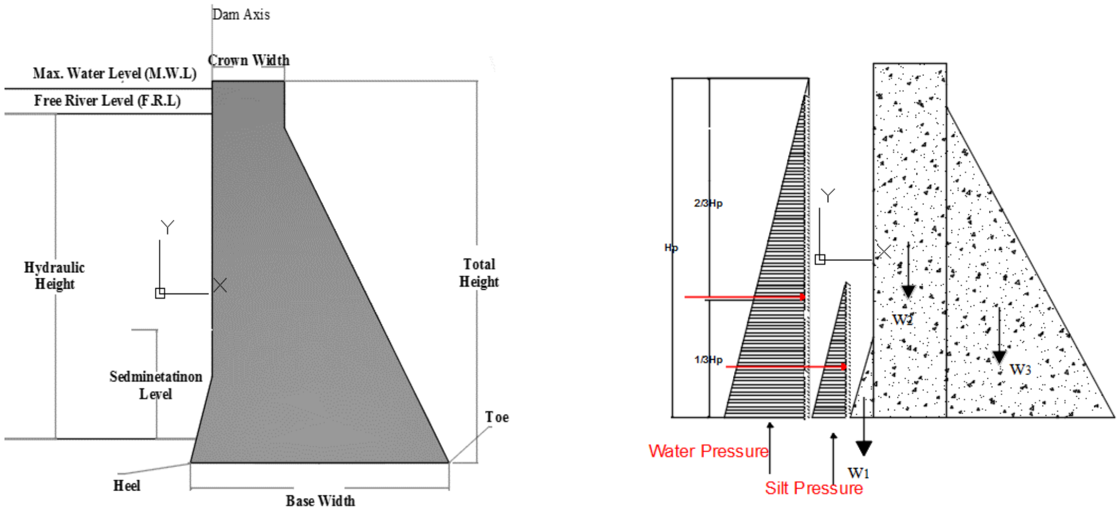
I. INTRODUCTION

Material cost is one of the major factors in the design and construction of massive structures. This issue becomes a major issue for structures such as gravity dams that relies on material weight. To reduce the construction cost, it is preferable to reduce the volume of concrete and all methods used for minimizing the volume of the concrete and the corresponding cost of the concrete are intended to achieve an optimal design.

It is important to understand the characteristics of the problem for selecting the appropriate optimization method [1, 2]. In a large construction project such as a dam, several goals and many constraints must be simultaneously satisfied to obtain an optimal solution. The parallel constraints applied when using a multi-objective function are typically conflicting and non-commensurable and must be simultaneously satisfied [3, 4]. Conversely, finding the objective function with logical-mathematical relations to solve an optimization problem for a large construction project using trial-and-error methods is difficult and sometimes unreachable. Several conventional optimization methods are proposed to reach an optimum solution. In a conventional method, such as Lagrange's or Newton's methods, the work domain should have first and second derivations [5] which would be potentially complex and, in some cases, hard to achieve. Another approach for optimization is heuristic algorithms. A heuristic optimization algorithm is an approach to problem-solving that employs a practical method to achieve a result, but which is not guaranteed to be optimal or perfect. Several methods such as evolutionary algorithms are used successfully in many optimization applications.

Evolutionary methods may be good choices to reach an optimum solution. Evolutionary competition describes the field of investigation that concerns all evolutionary algorithms and offers practical advantages to several optimization problems. The advantages include their simplicity of approach, robust response to changes in circumstances, and flexibility [6, 7]. Evolutionary methods have been used for many different applications such as slope stability design [8], retaining wall design [9], experimental data calibration [10], and structural failure prediction [11]. Previous studies have shown that these algorithms may be an appropriate tool for dam design solution such as topology optimization [12] and an efficient methodology for design of more efficient structures.

The most popular technique in evolutionary computation has been the genetic algorithm (GA). This method has a variety of capabilities, such as parallelism in solving problems, a complex fitness landscape, modifiability for different problems, application in multi-objective problems, and easy determination of the global optimum. In GA, each chromosome defines a unique numerical solution of the objective function and is subdivided into genes. Each gene is a single factor among the control factors. Each factor in the solution set corresponds to a gene on the chromosome. Choosing the chromosome with the best fit would be the global minimum and the best solution to the problem. However, the generation-producing methods, number of generations, selection methods, number of iterations and other factors can interfere with finding a satisfactory answer [13].



(a) (b)

Figure 1. Schematic of a dam body with: a) various levels definitions, and b) gravity load, water, and sedimentation pressures

This paper investigates the use of the genetic algorithm in the shape optimization of gravity dams under static and dynamic loads with a variety of shape parameters. Four existing dams are used as case studies in this research and optimization levels are reported for each case.

II. METHODOLOGY

A. Imposed loads

Dams are massive structures subjected to numerous loading scenarios in their lifetime. The main loading scenarios for a gravity dam are the weight of the dam mass, water pressure, sedimentation pressure, earthquake forces, wave pressure, ice pressure, wind forces, uplift forces, and thermal loads. A brief description of these loading scenarios is provided in this work.

Gravity load: The weight of the dam is a substantial load that provides stability to prevent overturning and sliding. The gravity load can be calculated by Equation 1.

$$W = \gamma_c . V \tag{1}$$

where W is the weight of each part; γ_c is the specific weight of the concrete, and V is the volume of each part in cubic meters. Geometry definition and various levels of dams as well as the gravity loads are presented in Figure 1a.

Water pressure: Water pressure is the main overturning force that would potentially destabilize the dam with increasing water depth. In the contrary to upstream, on the downstream side, the water pressure acts to stabilize the dam. Water pressure has a linear distribution and acts normal to the face of the dam. Pressure intensity is calculated in Equation 2, and the horizontal force because of the water pressure is calculated in Equation 3. Water pressure intensity in a dam is shown in Figure 1b.

$$P = \gamma_w h \tag{2}$$

where P is the pressure intensity; γ_w is the specific weight of the water, and h is the depth of the water at every point.

$$P_h = \frac{1}{2}\gamma_w . h^2 \tag{3}$$

Sedimentation pressures: Construction of the dam will cause the deposition of sedimentation behind the dam wall. The sedimentation pressures perform similarly to the water pressure. There are two methods to calculate sedimentation pressure: (1) the Rankine method, and (2) the experimental method.

The Rankine equations for calculating the sedimentation pressure in vertical and horizontal directions are as follows:

$$P_{SH} = \frac{\gamma'_s . h_s^2}{2} . \frac{1 - \sin\varphi}{1 + \sin\varphi} \tag{4}$$

$$P_{Sv} = \frac{\gamma'_s . h_s^2}{2} . \frac{1 + \sin\varphi}{1 - \sin\varphi} \tag{5}$$

where P_{SH} is the horizontal force because of upstream silt accumulation, γ'_s is the specific weight of the silt, Φ is the coefficient of friction for the sedimentation, h_s is the height of the sedimentation, and P_{SV} is vertical load acting on the heel of the dam as result of sedimentation weight. In this method, water pressure acting vertically on the heel should be separately calculated [14].

In the experimental method, the horizontal load of sedimentation is assumed to be similar to a liquid with a specific weight of 1360 Kg/m^3. For the vertical load calculation, the equivalent liquid-specific weight is assumed to be 1925 Kg/m^3 [15].

Uplift pressure: Seepage of water into the fractures and pores under the dam causes an upward pressure called uplift pressure which tends to destabilize the dam. The intensity of this pressure depends on the upstream and downstream water levels. Previous studies have shown that water cannot penetrate the entire area of the horizontal section of the dam. Therefore, uplift pressure should be applied to a part of the horizontal section. This area is calculated by applying the area coefficient, which ranges from 1/3 to 2/3. Standard dam design codes define the intensity distribution shape of the uplift pressure. The United States Bureau of Reclamation (USBR) provides a linear distribution for the calculation of uplift pressure [14].

Wave pressure: One of the considerable loads applied to a dam is wave pressure. Wave height depends on the wind speed, fetch length, and dam reservoir depth. Fetch length is defined as the horizontal distance over which wave-generating winds blow. Figure 2 illustrates the fetch length definition.

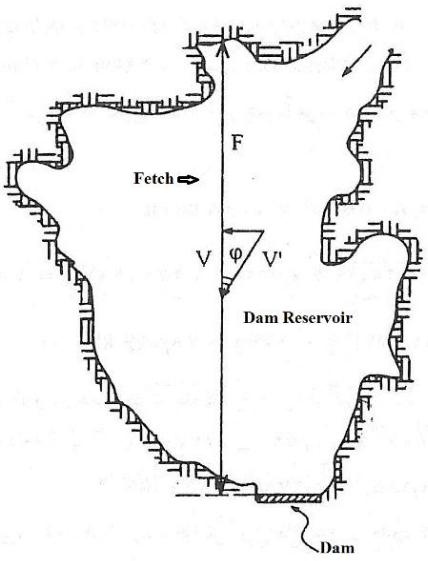


Figure 2. Wave Fetch in dam reservoir [15].

The maximum pressure of a wave is calculated by the following experimental equation [14]:

$$P_w = 2.4\gamma_w h_w \tag{6}$$

Where P_w is the wave pressure, and h_w is the wave height in meter. The maximum wave pressure intensity occurs $1/8h_w$ above the still-water level. The maximum wave force is calculated by Equation 7, and the maximum force effect point is located $3/8h_w$ above the still-water level. Figure 3 shows the maximum pressure distribution, maximum force and effective location on a dam section.

$$F_w = 2\gamma_w h_w^2 \tag{7}$$

The wave height is calculated by using the following equations:

$$\begin{aligned} &F \le 32Km \\ &h_w = 0.0322\sqrt{F.V} + 0.763 - 0.271F^{1/4} \end{aligned} \tag{8}$$

$$\begin{aligned} &F < 32Km \\ &h_w = 0.0322\sqrt{F.V} \end{aligned} \tag{9}$$

where F is the fetch length in *km*, and V is the wind velocity in *km/hr*.

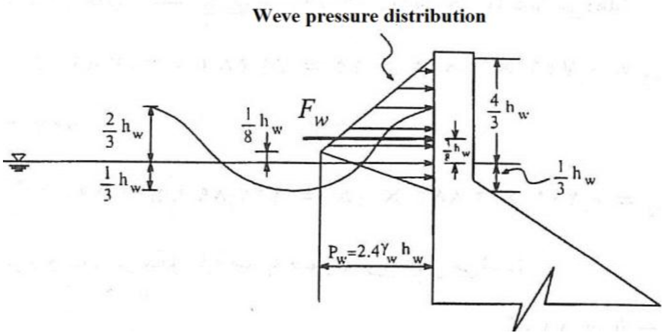


Figure 3. Wave forces on dam [15].

Horizontal earthquake load: Earthquake excitations are very significant for the heavy structures. There are several types of research regarding the seismic performance of the framed structures. In those studies, the lateral resistance of the structures was evaluated by employing two methods of applying earthquake excitation at the base or monotonic lateral loading along the structure height [16-23] or in bridge structures can be modeled as two inverse forces at inflection points [24]. To simulate earthquake loads in the experimental tests, a horizontal force applies at the tip of the cantilever type structures and this force increases step by step to evaluate seismic performance and structural damages at base [25]. Earthquake effect can be more pronounced for the gravity dams than the framed structures, because of larger mass and lack of redundancy of the structure. Several works have been performed to evaluate the performance of the dams due to earthquake loading [26-27]. Rayleigh and Love waves are important considerations for gravity dam earthquake loads. There are several methods to calculate earthquake horizontal force.

Block rocking analysis - in this method, the horizontal earthquake force on a dam is calculated by the following equation:

$$F_{eh} = a_h.w \tag{10}$$

where F_{eh} is the horizontal force acting on the dam due to the earthquake, a_h is the earthquake horizontal acceleration, and w is the dam weight.

Response spectra method the dam is considered in terms of oscillator modes with a single degree of freedom of vibration, which is weighted by target response spectrum curves. The modal responses are combined with the square root of the sum of squares (SRSS) or complete quadratic combination (CQC) method. For dams with a height greater than 100 meters, the horizontal force is calculated by using the following equation:

$$F_{hz} = 0.6C_f a_h w \tag{11}$$

where F_{ch} is the horizontal force on a dam with height z, a_h is the horizontal earthquake coefficient, w is the weight of the dam, and C_f is the coefficient obtained from the graph in Figure 4 [15].

The basic overturning moment is obtained by using the Equation 12.

$$M_z = 0.9C_m . a_h . Z_w \tag{12}$$

where C_m is the coefficient obtained from the graph in Figure 5, and Z_w is the distance from the base to the gravity center of the dam.

To obtain the earthquake load, other methods such as time history analyses, the pseudo-dynamic method, and the direct solution method could be used.

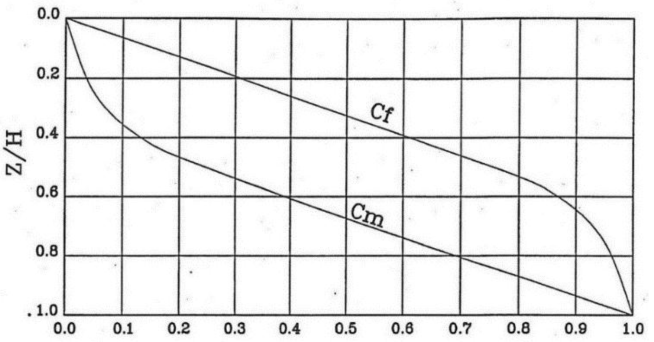


Figure 4. The graph for calculation of C_m and C_f in response spectrum method [15].

Vertical earthquake force: The earthquake force has another component that acts alongside with the gravity load in the vertical direction. It would contribute to dam stability if adds to gravity load or would destabilize the dam if act in an upward direction. It can be estimated with similar methods as horizontal earthquake forces using vertical earthquake coefficient instead of the horizontal coefficient.

Hydrodynamic force: Hydrodynamic force is the other important load on the dams which is the pressure imposed to the dam due to the horizontal acceleration of an earthquake on the reservoir. To determine this force, Zanger and Vestergaard methods could be used [28,29]. The USBR recommends Zanger method for obtaining the hydrodynamic pressure [14]. In this method, pressure distribution is assumed to have a curved shape and is obtained by Equation 12.

$$P_{ez} = C_e a_h \gamma_w \tag{12}$$

where C_e is calculated by:

$$C_e = \frac{C_m}{2}\left[\frac{Z}{h}(2-\frac{Z}{h}) + \sqrt{\frac{Z}{h}(2-\frac{Z}{h})}\right] \tag{13}$$

and C_m is obtained by equation 14.

$$C_m = 0.735\left(1-\frac{\varphi}{90^0}\right) \tag{14}$$

where Z is the maximum level of the dam reservoir, and h is the depth of the reservoir. Moreover, the C_m for different slopes upstream of a dam could be obtained from the graph in Fig. 5.

To obtain hydrodynamic pressure, Vestergaard provided the following equation [28]:

$$P = \frac{8a_h \gamma_w h}{\pi^2}\sum_{n=1,3,5,...}^{\infty}\frac{1}{n^{2.Cn}}\mathrm{Sin}\frac{n\pi Z}{2h} \tag{15}$$

$$C_n = \sqrt{1-\frac{16\gamma_w h}{n^2 g E_w Te^2}} \tag{16}$$

where a_h is the horizontal acceleration of the earthquake, γ_h is specific water weight, h is the depth of reservoir, E_w is the modulus of elasticity of the water, g is the gravitational acceleration, and T_e is the subjected earthquake period.

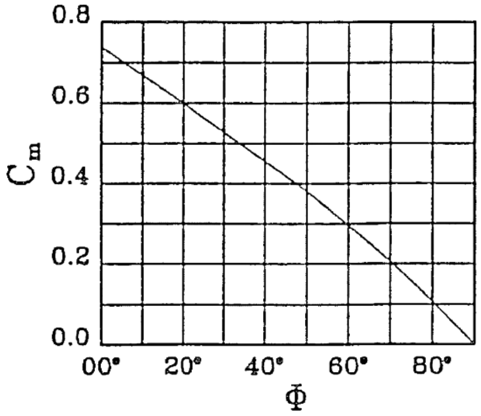


Figure 5. The graph for calculation C_m for hydraulic force in Zanger Method [15].

Other loads: There are several other loads such as temperature change due to cement hydration, wind load, and ice load that are not considered in this work.

B. *Load combination*

According to the requirement of USBR, three main load combinations are required for the design of a gravity dam. These three combinations are usual load combination, unusual load combination, and extreme load combination which are considered in this work [14].

III. CASE STUDIES ANALYSIS

Four case studies were selected for this research: Zavin Dam in Iran, Friant Dam in the United States [30], Koyna Dam in India, and Sariyar Dam in Turkey. For Sariyar dam, the earthquake accelerations are extracted from the seismic event happened close to the dam in 1999. About the Konya dam, it worth noting that the dam was hit by an earthquake with a magnitude of 6.3 in 1967. This earthquake caused some cracks at 66.5 meters above the base of the dam. Further study showed that the concrete was subjected to tensile stresses during the earthquake in the upstream side of the dam. Geometric and hydraulic, and design parameters of the dams are summarized in Table 1.

TABLE I. GEOMETRIC AND HYDRAULIC, AND DESIGN PARAMETERS OF THE CASE STUDIES.

	Zavin	Friant	Koyna	Sariyar
Maximum elevation (m)	42	97.2	103	96.5
Hydraulic elevation (m)	40	91.15	96.5	87.3
Crest width (m)	4.5	6.09	14.8	7
Base section length (m)	38.5	81.38	70	72
Silt elevation (m)	9	18.2	19.3	17
Wind speed (km/hr)	100	100	100	100
Fetch length (km)	10	20	20	20
Earthquake horizontal acceleration (g)	0.35	0.35	0.49	0.49
Earthquake vertical acceleration (g)	0.2	0.2	0.34	0.34
Compressive strength of concrete (MPa)	30	30	30	30
Concrete tensile strength (MPa)	0	0	0	0
Factor of Safety	1.5	1.5	1.5	1.5

- The special weights are assumed to be 2400 kg/m^3 for concrete, 1920 kg/m^3 for sedimentation, and 1000 kg/m^3 for water.

A. Objective Functions

The compressive and tensile stress at the toe and heel of the dams were calculated according to the USBR extreme load combination conditions. The safety factor for overturning the condition was considered to be 1.5. The objective function looks for the minimized value of the cross-sectional area of the dam by changing the upstream and downstream slopes of heel and toes. Constraints were the compressive stress of the dam at the downstream (toe), tension stress at upstream and demonstrative safety factor according to USBR code. Constraints were added by penalty function to the objective function. The toe and heel slopes are illustrated in Figure 6.

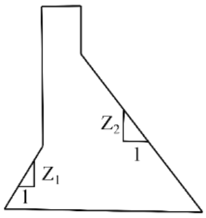


Figure 6. Optimization parameters

IV. RESULTS AND DISCUSSION

After performing the analyses, the optimized parameters of each dam were compared with the original constructing parameter values and the optimization level was obtained. For Zavin dam, optimized volume per unit of length for the dam section was 666.71 m^3. In contrast to the real volume is 760 m^3. This shows a 12.4% decrease in concrete volume. For Friant dam, the real volume of concrete of 4250 m^3 per unit of length was used. It can be understood that for a safety factor equal to 1.5, there is a 32% optimization, and for a safety factor equal to 2, there is 18.5% optimization for Friant Dam. By considering the height of the dam, a safety factor of 2 seems logical. For Sariyar dam, the real volume of concrete of 3365 m^3 per unit of length was used. It can be understood that for a safety factor equal to 1.5, there is a 34% optimization, and for a safety factor equal to 2, there is 18% optimization. For the Konya dam, the optimized volume of concrete per unit length was 2949 m^3 of concrete, which is an optimization of 3.5%. Compressive stress at the downstream, tensile stress at the upstream face and safety factor for optimal design are shown in Table 2.

A. Adoption of Optimization for Improving the Design of the Konya Dam.

The performance of the Konya dam during the earthquake was shown to be not satisfactory. The earthquake parameters are extracted from the captured seismic event of 1967. The values of compression and tension stresses are calculated for the existing design of the dam as shown in Table 3. The development of tensile stress in the dam is further confirmed by the observation of cracks during the 1967 earthquake.

To further increase the optimization, it was decided to add an extra parameter of crest width. The optimization searched for tree parameters of crest width, and toe and heel slopes. Compressive stress at the downstream, tensile stress at the upstream face and safety factor for optimal design are shown in Table 4.

TABLE II. DAM'S OPTIMAL PERFORMANCE.

	$\sigma_c\ (MPa)$	$\sigma_t\ (MPa)$	$F_{s\text{-}m}$	Z_1	Z_2	Optimized Volume*	Real Volume*	Reduction (%)
Zavin	0.951	0.153	1.5	0.004	0.692	667	760	12.4
Friant	2.304	0.686	1.5	0.2614	0.472	2889	4250	32
Friant	2.284	0.785	2.0	0.501	0.578	3482	4250	18.5
Sariyar	2.595	0.308	1.5	0.325	0.417	2205	3365	34
Sariyar	2.480	0.440	2.0	0.491	0.534	2738	3365	18
Konya	2.447	0.111	1.5	0.674	0.468	2949	3105	3.5

* Unit in m^3 of concrete per length of the dam.

TABLE III. COMPRESSION AND TENSION STRESSES FORMED IN KONYA DAM EXITING DESIGN.

	$\sigma_c\ (MPa)$	$\sigma_t\ (MPa)$	$F_{s\text{-}m}$
Konya	4.019	-1.245	1.5

TABLE IV. DAMS OPTIMAL PERFORMANCE

	$\sigma_c\ (MPa)$	$\sigma_t\ (MPa)$	$F_{s\text{-}m}$	Z_1	Z_2	Optimized Volume*	Real Volume*	Reduction (%)
Konya	2.384	0.487	1.5	0.507	0.689	2530	3105	18.5

* Unit in m^3 of concrete per length of the dam.

V. CONCLUSIONS

Application of genetic algorithm in the shape optimization of gravity dams under static and dynamic loads with a variety of shape parameters are investigated in this work. Four existing dams were used as case studies in this research and optimization criteria compared with the existing structures. Also, in case of Konya dam, the design of the dam was improved to mitigate the earthquake design issue using the optimization script. The following conclusions are made after performing this study:

1- An adaptive improvement is presented for the optimization of massive structures such as dams, and this procedure can be readily applied for optimization of concrete arch dams, gravity dams, soil dams or other massive structures. All the case studies showed various levels of optimization under the GA analysis;

2- GA analysis for Konya dam showed that for unusual loading with 1967 earthquake data, tensile restrictions for existing conditions were not satisfied, and tensile stresses occurred at the upstream face;

3- GA is a flexible method not only for optimization but also potentially for optimized design of the structures. The GA script was used to find optimum values for crest width of Konya dam successfully.

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