



From Spheroids To Globotoroids

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Abstract- A vast number of 3D objects surrounding us are spheroids, and in the case of our universe it certainly is more than evident. These objects are often geometrically similar, or more generally topologically homeomorphic, to spheres which over the past centuries have played an important role in mathematical sciences. Without them concept of the Riemann sphere, which is fundamental in defining the complex number and all the theories associated with it, would not exist. Despite its immense usefulness, the sphere alone cannot explain how object becomes a 3D spheroid in the first place. To address this, we examine a blend between torus and sphere topologies, none of which are homeomorphic to each other, and yet the combination produces some fascinating 3D objects. How is this possible? This paper presents such a possibility by examining results from recently proposed globotoroid theory.

Keywords- Spheres, Toroids, Wormholes

I. INTRODUCTION

The main difficulty of any 3D object transformation with the time is the space-time requirement. In the classical sense we need minimum 4 dimensions; 3 representing the object in space, and 1 for the time. This 4-dimensional framework opens a complexity that is not easy to model and visualize. For instance, if a ring torus was to grow into a horn torus in 3D, one would need to compile for each time segment an incremental 3D torus transformation. This is an awkward and daunting task which requires a lot of time to accomplish. What is an alternative?

Suppose the time variable can now be implicitly stated within the 3 space variables describing the torus objects. This would make the space-time requirement 3-dimensional, and as the time is implicit this space would play dynamic realizations of the 3 toroidal variables. That is, by visualizing these toroidal variables in the 3D space, one would experience a transformation of the ring torus as it grows into the horn torus. Nice, but how does one accomplish this?

The process of implicitly imbedding the time into space variables is one of the main properties given by the ordinary differential equations (ODE). The solutions of these equations form a dynamical space, usually the Euclidean space modeled by the real coordinate system R^n , where n is the space dimension. For the present discussion n will always be 3, and our space will always be R^3 , or 3D.

In the next section it is demonstrated how this idea works by using recently reported results from the globotoroid model [1]. In its basic form this model has 3 ODE equations with 3 parameters which produce a wide variety of spherical and toroidal solutions in R^3 . These solutions are trajectories that are analogous to spherical spirals, or scrolls, known as the loxodromes [2,3]. The exception in the present case is that here loxodromes have some unique features not previously reported. These will be revealed together with a surprising space-time structure in R^3 resulting from their dynamics. The paper concludes by discussing this simple equation and its modeling opportunities in physics, engineering and cosmology.

II. THE GLOBOTOROID MODEL AND ITS SPACE-TIME SOLUTIONS

A. The Spheroids

The model originally reported in [4,5,6], and subsequently termed as the globotoroid in [1], was slightly modified to produce the following 3 ODEs with 3 coefficients,

$$\begin{aligned}d X(t)/dt &= \omega Y(t) - A Z(t) X(t) \\d Y(t)/dt &= -\omega X(t) \\d Z(t)/dt &= -B + A[X(t)^2 + Y(t)^2 + 1]\end{aligned}\tag{1}$$

Here, t is the time, $X(t)$ and $Y(t)$ are referred to as the action, or orbital, space-time variables, the coefficient $\omega = 2\pi f$ is angular frequency with $f > 0$ being the orbital frequency. The space-time variable $Z(t)$ is the growth variable and is stimulated by the growth parameters $A, B > 0$.

To solve (1) it is recommended to use the Euler method, which was also applied in [1]. The stiff ODE solvers are not recommended for this equation as they tend to obscure the solutions.

Similarly, to what was reported in [1,6], (1) has only one possible equilibrium solution which is given by $X=0, Y=0$ and $B/A=1$. More precisely, this solution defines a singular manifold along Z axis, where for $Z < 0$ all the points are unstable, otherwise they are stable. As such, $B/A=1$ establishes a criticality condition at which the 3 space-time solutions form spheres, or spheroids. The spherical solutions are described by the loxodromic type trajectory whose scroll is determined by the orbital frequency f . For instance, the sphere solutions in Fig. 1A) are obtained for the set of coefficients $A=B=5, f=2\text{Hz}$ and the initial conditions $X_0=0.005, Y_0=0$ and $Z_0=-1$.

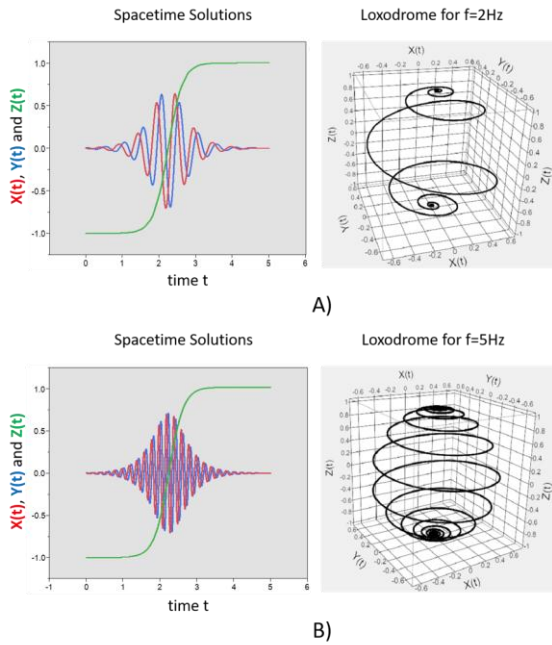


Figure 1. The frequency dependent spheroid solutions

Here, the space-time solutions are computed using the Euler method with the time increment $\Delta t=0.00005$. Fig. 1B) is evaluated using the same set of coefficients and initial conditions, with the exception that now $f=5\text{Hz}$. The loxodrome for $f=20\text{Hz}$ is depicted in Fig. 2A), and by continuing to increase f , the loxodrome gradually covers more of the sphere surface. Since the orbital frequency defines the spheroid together with its loxodrome, we also refer to f as the frequency of construction.

The question now is; what is inside the spheroid? By itself spheroid is hollow, however, by changing the initial conditions a set of concentrically nested spheroids fill its interior, Fig. 2B). For any given f , each spheroid is defined by a loxodrome which does not interact with other loxodromes in the nest. From the practical point of view this compact nest formation is highly unlikely. In natural processes it is difficult to maintain condition $A=B$ for a long time.

This setting, and its stability, closely resemble the phase space of the center type equilibrium in second-order ODEs. The difference is that in the latter case solutions are periodic, while for the concentric spheroid nest solutions are nonperiodic loxodromes.

Although the loxodrome exhibits spirals, it does not repeat with time. It begins at any unstable point on the singular manifold, and after coiling over a spheroid, it ends on the opposite side of the manifold. For (1) all the space points $[0,0, Z<0]$ are the unstable singularities, while $[0,0, Z\geq 0]$ are the stable ones.

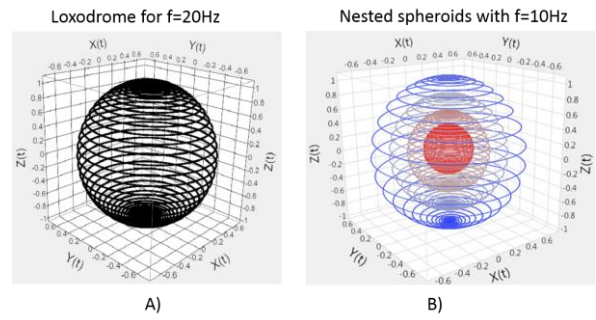


Figure 2. The spheroid cover and the concentric nest

B. The Growth Parameters A and B

Let's now consider the case when $A \neq B$, which implies that (1) is free from singularities. This case was fully investigated in [1,4,5,6]. Here, only the summary of the reported results is provided.

For $B/A < 1$ it was shown that the singular manifold turns into the slow linear manifold, which with time deflates the spheroid nest. In this case all loxodromes escape to $+\infty$ along the Z-axis, making this dynamics uneventful.

In contrast, for $B/A > 1$ dynamics becomes more interesting. Fig. 3 illustrates the space-time solutions for $B=5, A=4.5$ and the frequency of construction $f=10\text{Hz}$. The 3 cycles observed in 3 space-time solutions spend more time along the linear manifold than on the sphere. These solutions now form the periodic loxodrome, which in Fig. 3 also exhibits 3-cycles over 25 units of time, implying its period is about 8-time units, (note, when f is in Hz, the time unit is in seconds).

To recap, by letting $B > A$, the slow dynamics exerted along the linear manifold connects internally the spheroid poles and changes a nonperiodic loxodrome into a periodic one. This is the first time that any loxodrome is reported as having a periodic behavior, and together with being frequency dependent, the loxodrome sheds new light on spheroid dynamics.

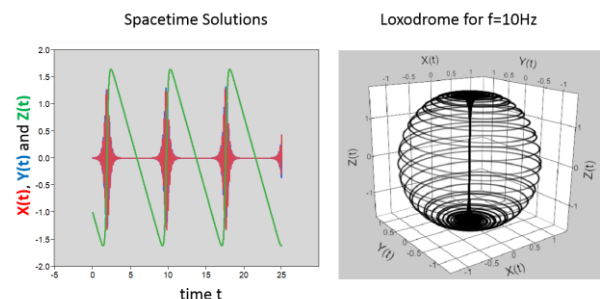


Figure 3. The periodic solutions and the loxodrome

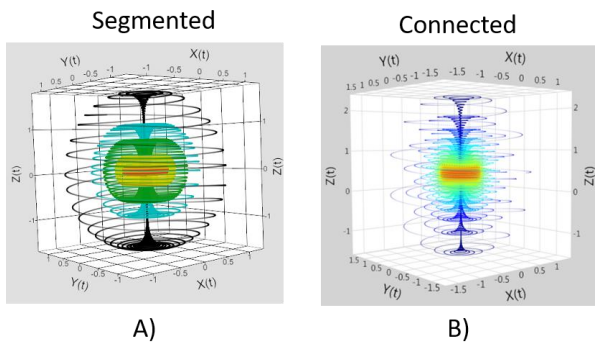


Figure 4. The globotoroid nest

What about all the concentrically nested spheroids in Fig. 2B)? They too get effected by transforming into tori. Collectively, spheroids and tori form a nest, which for visualization is presented as segmented in Fig. 4A). However, rather than being segmented, this nest is connected and allows the solutions $X(t)$, $Y(t)$ and $Z(t)$ to scroll over its constituents, Fig. 4B). The scroll is inflating if the inner core expands, and it is deflating if the core contracts. This behavior was previously noted in [1,5].

In conclusion, for $B > A$ the slow dynamics along the linear manifold changes the nest of concentric spheroids into the blend of connected spheroids and tori, hence, the name globotoroid. In addition, the linear manifold stimulates the globotoroid inflation, or deflation. For inflation the scroll is expanding, otherwise it is contracting.

C. The Wormhole

Why is this space-time object derived from the Einstein field equations, and also referred to as the Einstein-Rosen bridge [7,8], related to the globotoroids? To start, in both cases this bridge applies to curved spaces. The bridge forms a narrow opening which shortens the distance between maximally displaced curvatures, and in the present case this is between the spheroid poles. Moreover, this opening now allows loxodrome to reconnect.

However, one can argue that there is no wormhole in Fig. 4 since the spheroid with connected poles could be topologically interpreted as being a special case of torus. This interpretation would be incorrect, and the reasoning is that when we look at the nest in Fig. 4A), we can notice a distinct difference between the black spheroid and the rest of tori. This can also be noted in Figs. 4 and 6 in [5]. The difference results from the evolution of the red ring torus into the aqua horn torus, Fig. 4A). Once the horn torus is reached, the toroid structure wants to evolve further into the spindle torus. This, however, is dynamically impossible as it violates uniqueness of solutions at the spindle cross-points, Fig. 5. Thus, the solutions abandon the torus topology, and instead they elect the path of the linear manifold which drills the wormhole connecting the spheroid poles. As a result, this connection is always a straight line and lacks the usual toroidal curvature.

This newly imposed dynamic constraint reduces the torus hole into the wormhole and opens a path for loxodromes to

reconnect. During inflation the wormhole remarkably empowers transformation of tori into spheroids, while the empowerment is reversed during deflation.

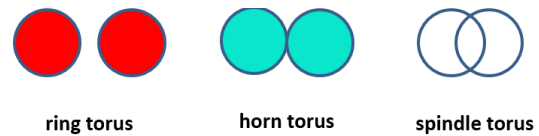


Figure 5. The cross-sections of the 3 different torus types

The wormholes have many interesting properties which will not be addressed here. However, there is one property that needs to be mentioned. For the inflationary globotoroid, the 3 space-time solutions are limited by the wormhole “choking” [1,6]. That is, as the globotoroid inflates it cannot grow indefinitely because the wormhole opening shrinks with time. Eventually, comes a point at which the 3 solutions, or the loxodrome, start to choke. At that point the limiting spheroid is reached, and inflation stops. The loxodrome circulates the limiting spheroid, while the choking sets off a soft indeterminism which makes the space-time solutions weakly chaotic [1,4,5,6]. When the globotoroid deflates, the solutions are moving away from the choking region, and are attracted by the core where the limit cycle resides [1].

III. DISCUSSION

With use of the simple 3-dimensional ODE it was demonstrated that its space-time solutions can describe spheroids in terms of the frequency regulated loxodromes. The behavior of spheroids and its loxodromes are further controlled by the growth parameters, A and B . When these parameters are equal the spheroids are organized in the concentric fashion, otherwise they inflate or deflate. For $A > B$, all spheroids deflate as loxodromes escape to infinity, while for $A < B$ interesting dynamics occurs. Here, the concentric spheroids turn into the globotoroid nest, which externally continues to support the spheroid geometry, while internally the toroid topology emerges. This nest organization is threaded by the wormhole which keeps the size of the nest limited and allows space-time solutions to scroll and form periodic loxodromes. When the scroll expands, loxodromes form the limiting spheroid. For the contracting scroll, the toroidal core with its limit cycle becomes a limiting set.

This diverse space-time behavior potentially offers many different applications of globotoroids in science and engineering. In physics it expands our understanding of angular momentum in the presence of wormholes. Fig. 6, entitled “The Worm”, illustrates how angular momentum expands a tiny worm within the wormhole. When the angular momentum is conserved, the tiny worm will spin faster than its expanded version on the sphere. In addition, if each yellow dot on the expanded worm represents a mass unit, then the total point mass of the tiny worm becomes very large. As a result, we now have a large mass, spinning very fast, inside the wormhole.

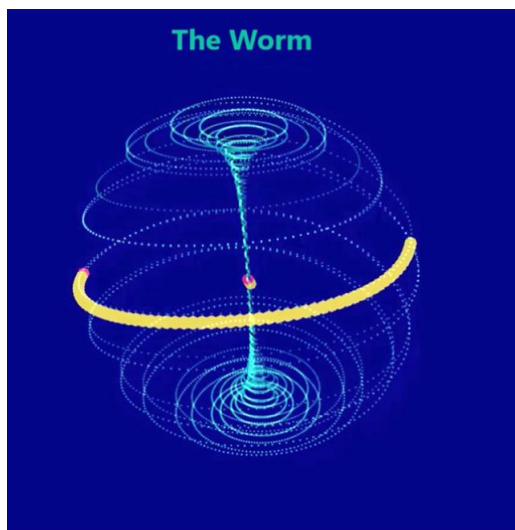


Figure 6. Angular momentum

Which brings us to cosmology and the subject of how energy and matter interact in our universe. Various celestial objects, including quasars, black holes, neutron stars and birth of young stars can be modeled using the globotoroid models. As a matter of fact, the doughnut theory of universe proposed by Alexi Starobinski and Yakov B. Zel'dovich in 1984 [9,10], treats the shape of universe as being a 3-dimensional torus. In this case analysis is still 4-dimensional and the model is not dynamic. The donut theory model can never produce spheroids as the wormhole is absent, and the underlined toroid topology is exclusive.

Another potential application of globotoroids is modeling of magnetic wormholes [11,12]. Here, the field of a magnetic source appears as an isolated magnetic monopole passes through the magnetostatic wormhole piercing the magnetically undetectable ferromagnetic sphere.

The globotoroid model is also used in mathematics to connect the subject of topological surgery with natural phenomena [13,14]. Some examples are, modeling extreme weather conditions, DNA recombination and DNA clamps, the electromagnetic near and far field pattern formations and the predator-prey type modeling in biological and economic systems. In conclusion, the globotoroid model has a right balance of topology and dynamics which in future may provide a more complete modeling platform for different phenomena in science and engineering.

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Nikola (Nick) Samardzija was born in Belgrade, Serbia, formerly part of Yugoslavia. After completing his high school education, he left Belgrade in pursuit of higher education. He obtained a bachelor's degree in electrical engineering from University of Bradford, and subsequently a master's degree in electrical engineering from University of Illinois. He also completed his PhD degree in chemical engineering at University of Leeds.

His professional calling led him to various research and data sciences positions at DuPont Co. and Emerson Electric Co. He published numerous papers and gave presentations at national and international conferences, primarily on the subject of nonlinear systems. He is also an inventor and has 10 patents.

In 2010 Dr. Samardzija founded an independent research initiative on exploring the subject of globotoroids. In 2011 this effort was named globotoroid.com after his web site www.globotoroid.com. Presently he is an independent researcher and manages all activities for globotoroid.com.