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On the Two-Phase Unrevised Simplex Method

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Abstract- In this paper, we deal with some forms of Two-Phase Unrevised Simplex Method (TPUSM) in solving Linear programming Problem (LPP), based on a given problem. The results from an algebraic calculation are checked, using the TORA software, a computing software and of reference in linear programming.

Keywords- *Two Phase-Method, Linear Programming Problem, Unrevised Simplex Method., TORA Software*

I. INTRODUCTION

The use of the unrevised simplex algorithm [1, 2, 3] we needs a set of m unit vectors which will form one identity matrix of order m. In some problems we may get less than m unit vectors. There may also be problems which do not contain any unit vector. In the last two cases we use the artificial basis technique to solve the given LPP. In these cases, one of the used methods in solving LPP is Two- phase method.

After adding artificial variables to the constraints of the LPP we get a set of m unit vectors. The latter constitute the initial basis. The process of eliminating artificial variables is performed in phase-I of the solution and phase-II is used to get an optimal solution. Since the solution of LPP is computed in two phases, it is called [1, 2, 3, 4] as Two-Phase Unrevised Simplex Method (TPUSM).

In Phase-I, the cost of the artificial variables is taken as -1 and those of other variables as zero. We get the new objective function as

Max Z* = ($-x_{n+1}$ $-x_{n+2}$ - … $-x_{n+m}$), where x_{n+1}, x_{n+2}, …, x_{n+m}, are artificial variables. The constraints being unchanged.

The problem then is solved by TPUSM. As each X_{n+i}, i = 1…m is non-negative, the maximum of new objective function is expected to be zero. Now three cases arise:

(1) Max Z* < 0 and at least one artificial vector appear in the optimum basis at a positive level.

(2) Max Z* = 0 and at least one artificial vector appears in the optimum basis at a zero level.

(3) Max Z* = 0 and no one artificial vector appears in the optimum basis.

In case (1) no feasible solution exists to the given LPP and hence we do not go to Phase-II to get optimal solution.

In case (2), we may or may not get an optimum basic feasible solution to the original problem. But we move to Phase-II to obtain an optimum basic feasible solution, if it exists.

In case (3) we get an initial basic feasible solution to the given LPP and then proceed to get optimal solution in Phase-II [1].

In Phase-II we assign the actual cost to the variables in the objective function and a zero cost to every artificial variable that appears in the basis at the zero level. This new objective function is now maximized by unrevised simplex method subject to the given constraints.

Unrevised Simplex method is applied to the modified unrevised simplex table obtained at the end of phase-I, until an optimum basic feasible solution has been attained. The artificial variables which are non-basic at the end of phase-I are removed. This Two- phase method has been studied since long [5, 6,7, 8, 9] and recently [3,10,11,12, 13,14].

In this paper, we give some forms of Two-Phase method and we make a comparison with the existing method. Using the TORA Software, we check the reliability of the results from an algebraic calculation.

II. PROBLEM, MATHEMATICAL FORMULATION AND METHODOLOGY

A. Problem

A jeweler wishes to manufacture 3 types of jewels J_1, J_2 *and* J_3. The unit price of these jewels are respectively 350, 200 and 400. The minimum composition of gold is 900 whereas the maximum compositions of silver and copper are respectively 1000 and 1200 in the appropriate units.

The following table indicates the minimum composition of each jewel:

TABLE I. MINIMUM COMPOSITION OF EACH JEWEL

	Gold	Silver	Copper
A	1	1	3
B	2	3	2
C	3	2	1

Determine the quantities of 3 Jewels to manufacture so that the total cost is maximum while respecting the data compositions.

B. Mathematical formulation

The mathematical formulation of the above problem is one of the followings LPP, where x_1, x_2 and x_3 are the numbers of A, B and C manufactured jewels, respectively:

$$\begin{cases} Max\ Z = 350\,x_1 + 200\,x_2 + 400\,x_3 \\ subject\ to \\ x_1 + 2\,x_2 + 3x_3 \geq 900 \\ x_1 + 3\,x_2 + 2x_3 \leq 1000 \\ 3\,x_1 + 2\,x_2 \leq 1200 \\ x_1, x_2, x_3 \geq 0, \end{cases} \quad (1)$$

$$\begin{cases} Max\ Z = 350\,x_1 + 200\,x_2 + 400\,x_3 \\ subject\ to \\ x_1 + 3\,x_2 + 2x_3 \leq 1000 \\ x_1 + 2\,x_2 + 3x_3 \geq 900 \\ 3\,x_1 + 2\,x_2 \leq 1200 \\ x_1, x_2, x_3 \geq 0, \end{cases} \quad (2)$$

$$\begin{cases} Max\ Z = 350\,x_1 + 200\,x_2 + 400\,x_3 \\ subject\ to \\ 3\,x_1 + 2\,x_2 \leq 1200 \\ x_1 + 2\,x_2 + 3x_3 \geq 900 \\ x_1 + 3\,x_2 + 2x_3 \leq 1000 \\ x_1, x_2, x_3 \geq 0, \end{cases} \quad (3)$$

$$\begin{cases} Max\ Z = 350\,x_1 + 200\,x_2 + 400\,x_3 \\ subject\ to \\ x_1 + 3\,x_2 + 2x_3 \leq 1000 \\ 3\,x_1 + 2\,x_2 \leq 1200 \\ x_1 + 2\,x_2 + 3x_3 \geq 900 \\ x_1, x_2, x_3 \geq 0, \end{cases} \quad (4)$$

$$\begin{cases} Max\ Z = 350\,x_1 + 200\,x_2 + 400\,x_3 \\ subject\ to \\ 3\,x_1 + 2\,x_2 \leq 1200 \\ x_1 + 3\,x_2 + 2x_3 \leq 1000 \\ x_1 + 2\,x_2 + 3x_3 \geq 900 \\ x_1, x_2, x_3 \geq 0, \end{cases} \quad (5)$$

C. Methodology

For the system "(1)", introducing surplus, artificial and slack variables we get k! differents systems, where,

k!= number of surplus variables + number of artificial variables + number of slack variable.

Here, k equals to 4 and we have 24 different systems (24 Si, i =1, …, 24).

The 4 permutations give us 4 cases:

Case 1:

In this case, the surplus variable is x_4. It follows that the initial basis will not contain that variable.

$$\begin{cases} Max\ Z = 350\,x_1 + 200\,x_2 + 400\,x_3 + 0x_4 - x_5 \\ +0x_6 + 0x_7 \\ subject\ to \\ x_1 + 2\,x_2 + 3x_3 - x_4 + x_5 = 900 \\ x_1 + 3\,x_2 + 2x_3 + x_6 = 1000 \\ 3\,x_1 + 2\,x_2 + x_7 = 1200 \\ x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0, \end{cases} \quad (S1)$$

For "(S1)", the variation of slack variables position leads to

$$\begin{cases} Max\ Z = 350\,x_1 + 200\,x_2 + 400\,x_3 + 0x_4 - x_5 \\ +0x_6 + 0x_7 \\ subject\ to \\ x_1 + 2\,x_2 + 3x_3 - x_4 + x_5 = 900 \\ x_1 + 3\,x_2 + 2x_3 + x_7 = 1000 \\ 3\,x_1 + 2\,x_2 + x_6 = 1200 \\ x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0, \end{cases} \quad (S2)$$

For "(S1)", the first variation of artificial variables position gives

$$\begin{cases} Max\ Z = 350\,x_1 + 200\,x_2 + 400\,x_3 + 0x_4 - x_5 \\ +0x_6 + 0x_7 \\ subject\ to \\ x_1 + 2\,x_2 + 3x_3 - x_4 + x_6 = 900 \\ x_1 + 3\,x_2 + 2x_3 + x_5 = 1000 \\ 3\,x_1 + 2\,x_2 + x_7 = 1200 \\ x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0, \end{cases} \quad (S3)$$

For "(S3)", the variation of slack variables position leads to

$$\begin{cases} Max\ Z = 350\,x_1 + 200\,x_2 + 400\,x_3 + 0x_4 - x_5 \\ +0x_6 + 0x_7 \\ subject\ to \\ x_1 + 2\,x_2 + 3x_3 - x_4 + x_6 = 900 \\ x_1 + 3\,x_2 + 2x_3 + x_7 = 1000 \\ 3\,x_1 + 2\,x_2 + x_5 = 1200 \\ x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0, \end{cases} \quad (S4)$$

For "(S1)", the second variation of artificial variables position gives

$$\begin{cases} Max\ Z = 350\,x_1 + 200\,x_2 + 400\,x_3 + 0x_4 - x_5 \\ +0x_6 + 0x_7 \\ subject\ to \\ x_1 + 2\,x_2 + 3x_3 - x_4 + x_7 = 900 \\ x_1 + 3\,x_2 + 2x_3 + x_5 = 1000 \\ 3\,x_1 + 2\,x_2 + x_6 = 1200 \\ x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0, \end{cases} \quad (S5)$$

For "(S5)", the variation of slack variables position leads to

$$\begin{cases} Max\ Z = 350\,x_1 + 200\,x_2 + \ 400\,x_3 + 0x_4 - x_5 \\ \quad +0x_6\ + 0x_7 \\ \quad subject\ to \\ x_1 + 2\,x_2 + 3x_3 - x_4 + x_7 = 900 \\ x_1 + 3\,x_2 + 2x_3 + x_6 = 1000 \\ 3\,x_1 + 2\,x_2 + x_5\ = 1200 \\ x_1, x_2, x_3, x_4\,, x_5\,, x_6\,, x_7\ \geq 0, \end{cases} \qquad \text{(S6)}$$

Case 2:

In this case, the surplus variable is x_5. It follows that the initial basis will not contain that variable.

$$\begin{cases} Max\ Z = 350\,x_1 + 200\,x_2 + \ 400\,x_3 + 0x_4 + 0x_5 \\ \quad -x_6\ + 0x_7 \\ \quad subject\ to \\ x_1 + 2\,x_2 + 3x_3 - x_5 + x_6 = 900 \\ x_1 + 3\,x_2 + 2x_3 + x_4 = 1000 \\ 3\,x_1 + 2\,x_2 + x_7\ = 1200 \\ x_1, x_2, x_3, x_4\,, x_5\,, x_6\,, x_7\ \geq 0, \end{cases} \qquad \text{(S7)}$$

For "(S7)", the variation of slack variables position leads to

$$\begin{cases} Max\ Z = 350\,x_1 + 200\,x_2 + \ 400\,x_3 + 0x_4 + 0x_5 \\ \quad -x_6\ + 0x_7 \\ \quad subject\ to \\ x_1 + 2\,x_2 + 3x_3 - x_5 + x_6 = 900 \\ x_1 + 3\,x_2 + 2x_3 + x_7 = 1000 \\ 3\,x_1 + 2\,x_2 + x_7\ = 1200 \\ x_1, x_2, x_3, x_4\,, x_5\,, x_6\,, x_7\ \geq 0, \end{cases} \qquad \text{(S8)}$$

For "(S7)", the first variation of artificial variables position gives

$$\begin{cases} Max\ Z = 350\,x_1 + 200\,x_2 + \ 400\,x_3 + 0x_4 + 0x_5 \\ \quad -x_6\ + 0x_7 \\ \quad subject\ to \\ x_1 + 2\,x_2 + 3x_3 - x_5 + x_7 = 900 \\ x_1 + 3\,x_2 + 2x_3 + x_4 = 1000 \\ 3\,x_1 + 2\,x_2 + x_6\ = 1200 \\ x_1, x_2, , x_4\,, x_5\,, x_6\,, x_7\ \geq 0, \end{cases} \qquad \text{(S9)}$$

For "(S9)", the variation of slack variables position variables leads to

$$\begin{cases} Max\ Z = 350\,x_1 + 200\,x_2 + \ 400\,x_3 + 0x_4 + 0x_5 \\ \quad -x_6\ + 0x_7 \\ \quad subject\ to \\ x_1 + 2\,x_2 + 3x_3 - x_5 + x_7 = 900 \\ x_1 + 3\,x_2 + 2x_3 + x_4 = 1000 \\ 3\,x_1 + 2\,x_2 + x_6\ = 1200 \\ x_1, x_2, x_3, x_4\,, x_5\,, x_6\,, x_7\ \geq 0, \end{cases} \qquad \text{(S10)}$$

For "(S7)", the second variation of artificial variables position gives

$$\begin{cases} Max\ Z = 350\,x_1 + 200\,x_2 + \ 400\,x_3 + 0x_4 + 0x_5 \\ \quad -x_6\ + 0x_7 \\ \quad subject\ to \\ x_1 + 2\,x_2 + 3x_3 - x_5 + x_4 = 900 \\ x_1 + 3\,x_2 + 2x_3 + x_6 = 1000 \\ 3\,x_1 + 2\,x_2 + x_7\ = 1200 \\ x_1, x_2, x_3, x_4\,, x_5\,, x_6\,, x_7\ \geq 0, \end{cases} \qquad \text{(S11)}$$

For "(S11)", the variation of slack variables position leads to

$$\begin{cases} Max\ Z = 350\,x_1 + 200\,x_2 + \ 400\,x_3 + 0x_4 + 0x_5 \\ \quad -x_6\ + 0x_7 \\ \quad subject\ to \\ x_1 + 2\,x_2 + 3x_3 - x_5 + x_4 = 900 \\ x_1 + 3\,x_2 + 2x_3 + x_7 = 1000 \\ 3\,x_1 + 2\,x_2 + x_6\ = 1200 \\ x_1, x_2, x_3, x_4\,, x_5\,, x_6\,, x_7\ \geq 0, \end{cases} \qquad \text{(S12)}$$

Case 3:

In this case, the surplus variable is x_6. It follows that the initial basis will not contain that variable

$$\begin{cases} Max\ Z = 350\,x_1 + 200\,x_2 + \ 400\,x_3 + 0x_4 + 0x_5 \\ \quad +0x_6\ - x_7 \\ \quad subject\ to \\ x_1 + 2\,x_2 + 3x_3 - x_6 + x_7 = 900 \\ x_1 + 3\,x_2 + 2x_3 + x_4 = 1000 \\ 3\,x_1 + 2\,x_2 + x_5\ = 1200 \\ x_1, x_2, x_3, x_4\,, x_5\,, x_6\,, x_7\ \geq 0, \end{cases} \qquad \text{(S13)}$$

For "(S13)", the variation of slack variables position leads to

$$\begin{cases} Max\ Z = 350\,x_1 + 200\,x_2 + \ 400\,x_3 + 0x_4 + 0x_5 \\ \quad +0x_6\ - x_7 \\ \quad subject\ to \\ x_1 + 2\,x_2 + 3x_3 - x_6 + x_7 = 900 \\ x_1 + 3\,x_2 + 2x_3 + x_5 = 1000 \\ 3\,x_1 + 2\,x_2 + x_4\ = 1200 \\ x_1, x_2, x_3, x_4\,, x_5\,, x_6\,, x_7\ \geq 0, \end{cases} \qquad \text{(S14)}$$

For "(S13)", the first variation of artificial variables position gives

$$\begin{cases} Max\ Z = 350\,x_1 + 200\,x_2 + \ 400\,x_3 + 0x_4 + 0x_5 \\ \quad +0x_6\ - x_7 \\ \quad subject\ to \\ x_1 + 2\,x_2 + 3x_3 - x_6 + x_5 = 900 \\ x_1 + 3\,x_2 + 2x_3 + x_4 = 1000 \\ 3\,x_1 + 2\,x_2 + x_7\ = 1200 \\ x_1, x_2, x_3, x_4\,, x_5\,, x_6\,, x_7\ \geq 0, \end{cases} \qquad \text{(S15)}$$

For "(S15)", the variation of slack variables position leads to

$$\begin{cases} Max\ Z = 350\,x_1 + 200\,x_2 + 400\,x_3 + 0x_4 + 0x_5 \\ \quad +0x_6 - x_7 \\ \quad subject\ to \\ x_1 + 2\,x_2 + 3x_3 - x_6 + x_5 = 900 \\ x_1 + 3\,x_2 + 2x_3 + x_7 = 1000 \\ 3\,x_1 + 2\,x_2 + x_4 = 1200 \\ x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0, \end{cases} \quad (S16)$$

For "(S13)", the second variation of artificial variables position gives

$$\begin{cases} Max\ Z = 350\,x_1 + 200\,x_2 + 400\,x_3 + 0x_4 + 0x_5 \\ \quad +0x_6 - x_7 \\ \quad subject\ to \\ x_1 + 2\,x_2 + 3x_3 - x_6 + x_4 = 900 \\ x_1 + 3\,x_2 + 2x_3 + x_5 = 1000 \\ 3\,x_1 + 2\,x_2 + x_7 = 1200 \\ x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0, \end{cases} \quad (S17)$$

For "(S17)", the variation of slack variables position leads to

$$\begin{cases} Max\ Z = 350\,x_1 + 200\,x_2 + 400\,x_3 + 0x_4 + 0x_5 \\ \quad +0x_6 - x_7 \\ \quad subject\ to \\ x_1 + 2\,x_2 + 3x_3 - x_6 + x_4 = 900 \\ x_1 + 3\,x_2 + 2x_3 + x_7 = 1000 \\ 3\,x_1 + 2\,x_2 + x_5 = 1200 \\ x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0, \end{cases} \quad (S18)$$

Case 4:

In this case, the surplus variable is x_7. It follows that the initial basis will not contain that variable.

$$\begin{cases} Max\ Z = 350\,x_1 + 200\,x_2 + 400\,x_3 + 0x_4 + 0x_5 \\ \quad -x_6 + x_7 \\ \quad subject\ to \\ x_1 + 2\,x_2 + 3x_3 + x_6 - x_7 = 900 \\ x_1 + 3\,x_2 + 2x_3 + x_4 = 1000 \\ 3\,x_1 + 2\,x_2 + x_5 = 1200 \\ x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0, \end{cases} \quad (S19)$$

For "(S19)", the variation of slack variables position leads to

$$\begin{cases} Max\ Z = 350\,x_1 + 200\,x_2 + 400\,x_3 + 0x_4 + 0x_5 \\ \quad -x_6 + x_7 \\ \quad subject\ to \\ x_1 + 2\,x_2 + 3x_3 + x_6 - x_7 = 900 \\ x_1 + 3\,x_2 + 2x_3 + x_5 = 1000 \\ 3\,x_1 + 2\,x_2 + x_4 = 1200 \\ x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0, \end{cases} \quad (S20)$$

. For "(S19)", the first variation of artificial variables position gives

$$\begin{cases} Max\ Z = 350\,x_1 + 200\,x_2 + 400\,x_3 + 0x_4 + 0x_5 \\ \quad -x_6 + x_7 \\ \quad subject\ to \\ x_1 + 2\,x_2 + 3x_3 + x_5 - x_7 = 900 \\ x_1 + 3\,x_2 + 2x_3 + x_4 = 1000 \\ 3\,x_1 + 2\,x_2 + x_6 = 1200 \\ x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0, \end{cases} \quad (S21)$$

For "(S19)", the variation of slack variables position leads to

$$\begin{cases} Max\ Z = 350\,x_1 + 200\,x_2 + 400\,x_3 + 0x_4 + 0x_5 \\ \quad -x_6 + x_7 \\ \quad subject\ to \\ x_1 + 2\,x_2 + 3x_3 + x_5 - x_7 = 900 \\ x_1 + 3\,x_2 + 2x_3 + x_6 = 1000 \\ 3\,x_1 + 2\,x_2 + x_4 = 1200 \\ x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0, \end{cases} \quad (S22)$$

For "(S19)", the second variation of artificial variables position gives

$$\begin{cases} Max\ Z = 350\,x_1 + 200\,x_2 + 400\,x_3 + 0x_4 + 0x_5 \\ \quad -x_6 + x_7 \\ \quad subject\ to \\ x_1 + 2\,x_2 + 3x_3 + x_4 - x_7 = 900 \\ x_1 + 3\,x_2 + 2x_3 + x_5 = 1000 \\ 3\,x_1 + 2\,x_2 + x_6 = 1200 \\ x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0, \end{cases} \quad (S23)$$

For "(S23)", the variation of slack variables position leads to

$$\begin{cases} Max\ Z = 350\,x_1 + 200\,x_2 + 400\,x_3 + 0x_4 + 0x_5 \\ \quad -x_6 + x_7 \\ \quad subject\ to \\ x_1 + 2\,x_2 + 3x_3 + x_4 - x_7 = 900 \\ x_1 + 3\,x_2 + 2x_3 + x_6 = 1000 \\ 3\,x_1 + 2\,x_2 + x_5 = 1200 \\ x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0, \end{cases} \quad (S24)$$

For all these 24 systems, we have two methods:

The first method [1,2,3] is developed with maximization and we get the optimality criterion when Z_j-$C_j \geq 0$. The follow –up will be given by Max Z^*.

For the second method, the optimality criterion is given by

$\hat{C} \leq 0$ (Maximization case) and $\hat{C} \geq 0$ (Minimization case), where

$$\hat{C} = C - C_B(A^B)^{-1}\text{A}. \quad (6)$$

C is the cost vector, A^B the matrix relating to the basic variables, C_B the cost vector related to the basis B and A is a matrix of order m. n.

If the optimality criterion is no t reached, we calculate

$$Min\left\{\frac{x_i}{y_{iq}}, y_{iq} > 0\right\} = \frac{x_p}{y_{pq}} \text{ (finite)} \quad (7)$$

Where $y_{iq} = (A^B)^{-1} A_q$, x_p the variable to be removed from the basis and $A_B$the sth column of A, x_i is given by the formula

x_B= $(A^B)^{-1}$b (b is the second member of the constraints equations).

The formula "(7)", is used for the maximization and the minimization.

III. RESULTS

We resolve "(S1)", using the two methods and we make a comparison.

A. First method

$$\begin{cases} Max\ Z = 350\, x_1 + 200\, x_2 + 400\, x_3 + 0x_4 - x_5 \\ \qquad +0x_6 + 0x_7 \\ subject\ to \\ x_1 + 2\, x_2 + 3x_3 - x_4 + x_5 = 900 \\ x_1 + 3\, x_2 + 2x_3 + x_6 = 1000 \\ 3\, x_1 + 2\, x_2 + x_7 = 1200 \\ x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0, \end{cases}$$

1) Phase I

In this phase we assign the cost -1 to the artificial variable x_5 and zero to those other variables. We get

$$\begin{cases} Max\ Z = -x_5 \\ subject\ to \\ x_1 + 2\, x_2 + 3x_3 - x_4 + x_5 = 900 \\ x_1 + 3\, x_2 + 2x_3 + x_6 = 1000 \\ 3\, x_1 + 2\, x_2 + x_7 = 1200 \\ x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0, \end{cases}$$

TABLE II. STARTING TABLE

B	C_B	X_B	x_1	x_2	x_3	x_4	x_5	x_6	x_7
x_5	-1	900	1	2	3PE	-1	1	0	0
x_6	0	1000	1	3	2	0	0	1	0
x_7	0	1200	3	2	0	0	0	0	1
Z_j-C_j		-900	-1	-2	-3	1	0	0	0

PE is pivot element. Variable x_3 is introduced and x_5 is removed from the basis.

TABLE III.

B	C_B	X_B	x_1	x_2	x_3	x_4	x_5	x_6	x_7
x_3	0	300	$\frac{1}{3}$	$\frac{2}{3}$	1	$-\frac{1}{3}$	1	0	0
x_6	0	400	$\frac{1}{3}$	$\frac{5}{3}$	0	$\frac{2}{3}$	$-\frac{2}{3}$	1	0
x_7	0	1200	3	2	0	0	0	0	1
Z_j-C_j		0	0	0	0	0	1	0	0

Since all Z_j-$C_j \geq 0$ and Max. Z* = 0 we get an optimal solution. This gives a basic feasible solution to the original LPP which will be used in phase II to get optimal basic feasible solution.

2) Phase II

Here the actual costs are taken for the variables except the artificial variable which is assigned zero value. We now use the following objective function :

Max.Z= $350\, x_1 + 200\, x_2 + 400\, x_3 + 0x_4 + 0x_6 + 0x_7$.

TABLE IV.

B	C_B	X_B	x_1	x_2	x_3	x_4	x_6	x_7
x_3	400	300	$\frac{1}{3}$	$\frac{2}{3}$	1	$-\frac{1}{3}$	0	0
x_6	0	400	$\frac{1}{3}$	$\frac{5}{3}$	0	$\frac{2}{3}$	1	0
x_7	0	1200	3PE	2	0	0	0	1
Z_j-C_j		120000	$-\frac{650}{3}$	$\frac{200}{3}$	0	$-\frac{400}{3}$	0	0

Variable x_1 is introduced and x_7 is removed from the basis.

TABLE V.

B	C_B	X_B	x_1	x_2	x_3	x_4	x_6	x_7
x_3	400	$\frac{500}{3}$	0	$\frac{4}{9}$	1	$-\frac{1}{3}$	0	$-\frac{1}{9}$
x_6	0	$\frac{800}{3}$	0	$\frac{13}{9}$	0	$\frac{2}{3}$PE	1	$-\frac{1}{9}$
x_1	350	400	1	$\frac{2}{3}$	0	0	0	$\frac{1}{3}$
Z_j-C_j		120000	0	$\frac{1900}{9}$	0	$-\frac{400}{3}$	0	$\frac{650}{9}$

Variable x_4 is introduced and x_6 is removed from the basis.

TABLE VI.

B	C_B	X_B	x_1	x_2	x_3	x_4	x_6	x_7
x_3	400	300	0	$\frac{7}{6}$	1	0	$\frac{1}{2}$	$-\frac{1}{6}$
x_4	0	400	0	$\frac{13}{6}$	0	1	$\frac{3}{3}$	$-\frac{1}{6}$
x_1	350	400	1	$\frac{2}{3}$	0	0	0	$\frac{1}{3}$
Z_j-C_j		260000	0	500	0	0	200	50

Since all Z_j-$C_j \geq 0$, an optimum basic feasible solution is obtained.

Hence the optimum solution is:

$x_1 = 400$, $x_2 = 0$, $x_3 = 300$ and Max.Z= 260 000.

B. Second method

We take the basis

B $=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

Using the formula "(6)", we get

$\hat{C} = (1, 2, 3, -1, 0, 0, 0)$ and the condition $\hat{C} \leq 0$ is not reached. By the formula "(7)", the new basis is given by the columns corresponding to x_3 , x_6 and x_7 in matrix A. We get

$\hat{C} = (0, 0, 0,0, -1, 0, 0)$.

After getting pivot element, we use the following formula for the transformations [13, 15]:

$$L_i \, (L_i \neq L_{pivot}) \rightarrow L_i - (\text{element of } L_i \, \frac{1}{PE} L_{pivot}) \, , \qquad (8)$$

where L_{pivot} is the pivot row.

We can calculate the ratio:

$$\text{Ratio} = \frac{element\ of\ the\ column\ b}{element\ (not\ empty)\ of\ pivot\ column} \qquad (9)$$

and we take the minimum. This gives us the variable to remove from the basis. This is equivalent to the formula "(7)".

TABLE VII. PHASE I

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	b
x_5	1	2	3PE	-1	1	0	0	900
x_6	1	3	2	0	0	1	0	1000
x_7	3	2	0	0	0	0	1	1200
$\hat{C}$	1	2	3	-1	0	0	0	
x_3	$\frac{1}{3}$	$\frac{2}{3}$	1	$-\frac{1}{3}$	1	0	0	300
x_6	$\frac{1}{3}$	$\frac{5}{3}$	0	$\frac{2}{3}$	$-\frac{2}{3}$	1	0	400
x_7	3	2	0	0	0	0	1	1200
$\hat{C}$	0	0	0	0	-1	0	0	

Since all $\hat{C} \leq 0$ and Max. Z* = 0 we get an optimal solution. This gives a basic feasible solution to the original LPP which will be used in phase II to get optimal basic feasible solution.

Here the actual costs are taken for the variables except the artificial variable which is assigned zero value. We now use the following objective function:

Max.Z= $350\, x_1 + 200\, x_2 + 400\, x_3 + 0x_4 + 0x_6 + 0x_7$.

TABLE VIII. PHASE II

	x_1	x_2	x_3	x_4	x_6	x_7	b
x_3	$\frac{1}{3}$	$\frac{2}{3}$	1	$-\frac{1}{3}$	0	0	300
x_6	$\frac{1}{3}$	$\frac{5}{3}$	0	$\frac{2}{3}$	1	0	400
x_7	3PE	2	0	0	0	1	1200
$\hat{C}$	$\frac{650}{3}$	$-\frac{200}{3}$	0	$\frac{400}{3}$	0	0	
x_3	0	$\frac{4}{9}$	1	$-\frac{1}{3}$	0	$-\frac{1}{9}$	$\frac{500}{3}$
x_6	0	$\frac{13}{9}$	0	$\frac{2}{3}$PE	1	$-\frac{1}{9}$	$\frac{800}{3}$
x_1	1	$\frac{2}{3}$	0	0	0	$\frac{1}{3}$	400
$\hat{C}$	0	$-\frac{1900}{9}$	0	$\frac{400}{3}$	0	$-\frac{650}{9}$	
x_3	0	$\frac{7}{6}$	1	0	$\frac{1}{2}$	$-\frac{1}{6}$	300
x_4	0	$\frac{13}{6}$	0	1	$\frac{3}{3}$	$-\frac{1}{6}$	400
x_1	1	$\frac{2}{3}$	0	0	0	$\frac{1}{3}$	400
$\hat{C}$	0	-500	0	0	-200	-50	

Since all Z_j-$C_j \geq 0$, an optimum basic feasible solution is obtained.

Hence the optimum solution is

$x_1 = 400$, $x_2 = 0$, $x_3 = 300$ and Max.Z= 260 000.

C. Illustration Using TORA Software

With TORA software, the solution is obtained after the iteration 5:

Phase 1 (Iter 1								
Basic	x1	x2	x3	Sx4	Rx5	sx6	sx7	Solution
z (min)	1,00	2,00	3,00	-1,00	0,00	0,00	0,00	900,00
Rx5	1,00	2,00	3,00	-1,00	1,00	0,00	0,00	900,00
sx6	1,00	3,00	2,00	0,00	0,00	1,00	0,00	1000,00
sx7	3,00	2,00	0,00	0,00	0,00	0,00	1,00	1200,00
Lower Bound	0,00	0,00	0,00					
Upper Bound	infinity	infinity	infinity					
Unrestr'd (y/n)?	n	n	n					
Phase 1 (Iter 2								
Basic	x1	x2	x3	Sx4	Rx5	sx6	sx7	Solution
z (min)	0,00	0,00	0,00	0,00	-1,00	0,00	0,00	0,00
x3	0,33	0,67	1,00	-0,33	0,33	0,00	0,00	300,00
sx6	0,33	1,67	0,00	0,67	-0,67	1,00	0,00	400,00
sx7	3,00	2,00	0,00	0,00	0,00	0,00	1,00	1200,00

Figure 1. Phase I (Iteration 1& 2)

Phase 2 (Iter 3								
Basic	x1	x2	x3	Sx4	Rx5	sx6	sx7	Solution
z (max)	-216,67	66,67	0,00	-133,33	blocked	0,00	0,00	120000,00
x3	0,33	0,67	1,00	-0,33	0,33	0,00	0,00	300,00
sx6	0,33	1,67	0,00	0,67	-0,67	1,00	0,00	400,00
sx7	3,00	2,00	0,00	0,00	0,00	0,00	1,00	1200,00
Lower Bound	0,00	0,00	0,00					
Upper Bound	infinity	infinity	infinity					
Unrestr'd (y/n)?	n	n	n					
Phase 2 (Iter 4								
Basic	x1	x2	x3	Sx4	Rx5	sx6	sx7	Solution
z (max)	0,00	211,11	0,00	-133,33	blocked	0,00	72,22	206666,67
x3	0,00	0,44	1,00	-0,33	0,33	0,00	-0,11	166,67
sx6	0,00	1,44	0,00	0,67	-0,67	1,00	-0,11	266,67
x1	1,00	0,67	0,00	0,00	0,00	0,00	0,33	400,00

Figure 2. Phase II (Iteration 3& 4)

Phase 2 (Iter 5								
Basic	x1	x2	x3	Sx4	Rx5	sx6	sx7	Solution
z (max)	0,00	500,00	0,00	0,00	blocked	200,00	50,00	260000,00
x3	0,00	1,17	1,00	0,00	0,00	0,50	-0,17	300,00
Sx4	0,00	2,17	0,00	1,00	-1,00	1,50	-0,17	400,00
x1	1,00	0,67	0,00	0,00	0,00	0,00	0,33	400,00

Figure 3. Phase II (Iteration 5)

1) Illustration for "(2)"

Phase 1 (Iter 1								
Basic	x1	x2	x3	Sx4	sx5	Rx6	sx7	Solution
z (min)	1,00	2,00	3,00	-1,00	0,00	0,00	0,00	900,00
sx5	1,00	3,00	2,00	0,00	1,00	0,00	0,00	1000,00
Rx6	1,00	2,00	3,00	-1,00	0,00	1,00	0,00	900,00
sx7	3,00	2,00	0,00	0,00	0,00	0,00	1,00	1200,00
Lower Bound	0,00	0,00	0,00					
Upper Bound	infinity	infinity	infinity					
Unrestr'd (y/n)?	n	n	n					
Phase 1 (Iter 2								
Basic	x1	x2	x3	Sx4	sx5	Rx6	sx7	Solution
z (min)	0,00	0,00	0,00	0,00	0,00	-1,00	0,00	0,00
sx5	0,33	1,67	0,00	0,67	1,00	-0,67	0,00	400,00
x3	0,33	0,67	1,00	-0,33	0,00	0,33	0,00	300,00
sx7	3,00	2,00	0,00	0,00	0,00	0,00	1,00	1200,00
Lower Bound	0,00	0,00	0,00					
Upper Bound	infinity	infinity	infinity					
Unrestr'd (y/n)?	n	n	n					
Phase 2 (Iter 3								
Basic	x1	x2	x3	Sx4	sx5	Rx6	sx7	Solution
z (max)	-216,67	66,67	0,00	-133,33	0,00	blocked	0,00	120000,00
sx5	0,33	1,67	0,00	0,67	1,00	-0,67	0,00	400,00
x3	0,33	0,67	1,00	-0,33	0,00	0,33	0,00	300,00
sx7	3,00	2,00	0,00	0,00	0,00	0,00	1,00	1200,00

Figure 4. Phase I (Iteration 1&2) and Phase 2 (Iteration 3)

Phase 2 (Iter 4								
Basic	x1	x2	x3	Sx4	sx5	Rx6	sx7	Solution
z (max)	0,00	211,11	0,00	-133,33	0,00	blocked	72,22	206666,67
sx5	0,00	1,44	0,00	0,67	1,00	-0,67	-0,11	266,67
x3	0,00	0,44	1,00	-0,33	0,00	0,33	-0,11	166,67
x1	1,00	0,67	0,00	0,00	0,00	0,00	0,33	400,00
Lower Bound	0,00	0,00	0,00					
Upper Bound	infinity	infinity	infinity					
Unrestr'd (y/n)?	n	n	n					
Phase 2 (Iter 5								
Basic	x1	x2	x3	Sx4	sx5	Rx6	sx7	Solution
z (max)	0,00	500,00	0,00	0,00	200,00	blocked	50,00	260000,00
Sx4	0,00	2,17	0,00	1,00	1,50	-1,00	-0,17	400,00
x3	0,00	1,17	1,00	0,00	0,50	0,00	-0,17	300,00
x1	1,00	0,67	0,00	0,00	0,00	0,00	0,33	400,00

Figure 5. Phase 2 (Iteration 4 &5)

2) Illustration for "(3)".

Phase 1 (Iter 1								
Basic	x1	x2	x3	Sx4	sx5	Rx6	sx7	Solution
z (min)	1,00	2,00	3,00	-1,00	0,00	0,00	0,00	900,00
sx5	3,00	2,00	0,00	0,00	1,00	0,00	0,00	1200,00
Rx6	1,00	2,00	3,00	-1,00	0,00	1,00	0,00	900,00
sx7	1,00	3,00	2,00	0,00	0,00	0,00	1,00	1000,00
Lower Bound	0,00	0,00	0,00					
Upper Bound	infinity	infinity	infinity					
Unrestr'd (y/n)?	n	n	n					
Phase 1 (Iter 2								
Basic	x1	x2	x3	Sx4	sx5	Rx6	sx7	Solution
z (min)	0,00	0,00	0,00	0,00	0,00	-1,00	0,00	0,00
sx5	3,00	2,00	0,00	0,00	1,00	0,00	0,00	1200,00
x3	0,33	0,67	1,00	-0,33	0,00	0,33	0,00	300,00
sx7	0,33	1,67	0,00	0,67	0,00	-0,67	1,00	400,00
Lower Bound	0,00	0,00	0,00					
Upper Bound	infinity	infinity	infinity					
Unrestr'd (y/n)?	n	n	n					
Phase 2 (Iter 3								
Basic	x1	x2	x3	Sx4	sx5	Rx6	sx7	Solution
z (max)	-216,67	66,67	0,00	-133,33	0,00	blocked	0,00	120000,00
sx5	3,00	2,00	0,00	0,00	1,00	0,00	0,00	1200,00
x3	0,33	0,67	1,00	-0,33	0,00	0,33	0,00	300,00
sx7	0,33	1,67	0,00	0,67	0,00	-0,67	1,00	400,00

Figure 6. Phase I (Iteration 1&2) and Phase 2 (Iteration 3)

Phase 2 (Iter 4								
Basic	x1	x2	x3	Sx4	sx5	Rx6	sx7	Solution
z (max)	0,00	211,11	0,00	-133,33	72,22	blocked	0,00	206666,67
x1	1,00	0,67	0,00	0,00	0,33	0,00	0,00	400,00
x3	0,00	0,44	1,00	-0,33	-0,11	0,33	0,00	166,67
sx7	0,00	1,44	0,00	0,67	-0,11	-0,67	1,00	266,67
Lower Bound	0,00	0,00	0,00					
Upper Bound	infinity	infinity	infinity					
Unrestr'd (y/n)?	n	n	n					
Phase 2 (Iter 5								
Basic	x1	x2	x3	Sx4	sx5	Rx6	sx7	Solution
z (max)	0,00	500,00	0,00	0,00	50,00	blocked	200,00	260000,00
x1	1,00	0,67	0,00	0,00	0,33	0,00	0,00	400,00
x3	0,00	1,17	1,00	0,00	-0,17	0,00	0,50	300,00
Sx4	0,00	2,17	0,00	1,00	-0,17	-1,00	1,50	400,00

Figure 7. Phase 2 (Iteration 4 &5)

3) *Illustration for "(4)".*

Phase 1 (Iter 1								
Basic	x1	x2	x3	Sx4	sx5	sx6	Rx7	Solution
z (min)	1,00	2,00	3,00	-1,00	0,00	0,00	0,00	900,00
sx5	1,00	3,00	2,00	0,00	1,00	0,00	0,00	1000,00
sx6	3,00	2,00	0,00	0,00	0,00	1,00	0,00	1200,00
Rx7	1,00	2,00	3,00	-1,00	0,00	0,00	1,00	900,00
Lower Bound	0,00	0,00	0,00					
Upper Bound	infinity	infinity	infinity					
Unrestr'd (y/n)?	n	n	n					
Phase 1 (Iter 2								
Basic	x1	x2	x3	Sx4	sx5	sx6	Rx7	Solution
z (min)	0,00	0,00	0,00	0,00	0,00	0,00	-1,00	0,00
sx5	0,33	1,67	0,00	0,67	1,00	0,00	-0,67	400,00
sx6	3,00	2,00	0,00	0,00	0,00	1,00	0,00	1200,00
x3	0,33	0,67	1,00	-0,33	0,00	0,00	0,33	300,00
Lower Bound	0,00	0,00	0,00					
Upper Bound	infinity	infinity	infinity					
Unrestr'd (y/n)?	n	n	n					
Phase 2 (Iter 3								
Basic	x1	x2	x3	Sx4	sx5	sx6	Rx7	Solution
z (max)	-216,67	66,67	0,00	-133,33	0,00	0,00	blocked	120000,00
sx5	0,33	1,67	0,00	0,67	1,00	0,00	-0,67	400,00
sx6	3,00	2,00	0,00	0,00	0,00	1,00	0,00	1200,00
x3	0,33	0,67	1,00	-0,33	0,00	0,00	0,33	300,00

Figure 8. Phase I (Iteration 1&2) and Phase 2 (Iteration 3)

Phase 2 (Iter 4								
Basic	x1	x2	x3	Sx4	sx5	sx6	Rx7	Solution
z (max)	0,00	211,11	0,00	-133,33	0,00	72,22	blocked	206666,67
sx5	0,00	1,44	0,00	0,67	1,00	-0,11	-0,67	266,67
x1	1,00	0,67	0,00	0,00	0,00	0,33	0,00	400,00
x3	0,00	0,44	1,00	-0,33	0,00	-0,11	0,33	166,67
Lower Bound	0,00	0,00	0,00					
Upper Bound	infinity	infinity	infinity					
Unrestr'd (y/n)?	n	n	n					
Phase 2 (Iter 5								
Basic	x1	x2	x3	Sx4	sx5	sx6	Rx7	Solution
z (max)	0,00	500,00	0,00	0,00	200,00	50,00	blocked	260000,00
Sx4	0,00	2,17	0,00	1,00	1,50	-0,17	-1,00	400,00
x1	1,00	0,67	0,00	0,00	0,00	0,33	0,00	400,00
x3	0,00	1,17	1,00	0,00	0,50	-0,17	0,00	300,00

Figure 9. Phase 2 (Iteration 4 &5)

4) *Illustration for "(5)".*

Phase 1 (Iter 1								
Basic	x1	x2	x3	Sx4	sx5	sx6	Rx7	Solution
z (min)	1,00	2,00	3,00	-1,00	0,00	0,00	0,00	900,00
sx5	3,00	2,00	0,00	0,00	1,00	0,00	0,00	1200,00
sx6	1,00	3,00	2,00	0,00	0,00	1,00	0,00	1000,00
Rx7	1,00	2,00	3,00	-1,00	0,00	0,00	1,00	900,00
Lower Bound	0,00	0,00	0,00					
Upper Bound	infinity	infinity	infinity					
Unrestr'd (y/n)?	n	n	n					
Phase 1 (Iter 2								
Basic	x1	x2	x3	Sx4	sx5	sx6	Rx7	Solution
z (min)	0,00	0,00	0,00	0,00	0,00	0,00	-1,00	0,00
sx5	3,00	2,00	0,00	0,00	1,00	0,00	0,00	1200,00
sx6	0,33	1,67	0,00	0,67	0,00	1,00	-0,67	400,00
x3	0,33	0,67	1,00	-0,33	0,00	0,00	0,33	300,00
Lower Bound	0,00	0,00	0,00					
Upper Bound	infinity	infinity	infinity					
Unrestr'd (y/n)?	n	n	n					
Phase 2 (Iter 3								
Basic	x1	x2	x3	Sx4	sx5	sx6	Rx7	Solution
z (max)	-216,67	66,67	0,00	-133,33	0,00	0,00	blocked	120000,00
sx5	3,00	2,00	0,00	0,00	1,00	0,00	0,00	1200,00
sx6	0,33	1,67	0,00	0,67	0,00	1,00	-0,67	400,00
x3	0,33	0,67	1,00	-0,33	0,00	0,00	0,33	300,00

Figure 10. Phase I (Iteration 1&2) and Phase 2 (Iteration 3)

Phase 2 (Iter 4								
Basic	x1	x2	x3	Sx4	sx5	sx6	Rx7	Solution
z (max)	0,00	211,11	0,00	-133,33	72,22	0,00	blocked	206666,67
x1	1,00	0,67	0,00	0,00	0,33	0,00	0,00	400,00
sx6	0,00	1,44	0,00	0,67	-0,11	1,00	-0,67	266,67
x3	0,00	0,44	1,00	-0,33	-0,11	0,00	0,33	166,67
Lower Bound	0,00	0,00	0,00					
Upper Bound	infinity	infinity	infinity					
Unrestr'd (y/n)?	n	n	n					
Phase 2 (Iter 5								
Basic	x1	x2	x3	Sx4	sx5	sx6	Rx7	Solution
z (max)	0,00	500,00	0,00	0,00	50,00	200,00	blocked	260000,00
x1	1,00	0,67	0,00	0,00	0,33	0,00	0,00	400,00
Sx4	0,00	2,17	0,00	1,00	-0,17	1,50	-1,00	400,00
x3	0,00	1,17	1,00	0,00	-0,17	0,50	0,00	300,00

Figure 11. Phase 2 (Iteration 4 &5)

IV. CONCLUSION AND FUTURE WORK

In this paper, a new form of TPUSM is proposed and it is very easy to understand and provides better result in comparison to the existing method available in the literature. It is general because is used for maximizing objective function and minimizing objective problem without any transformation. The TORA software is the suitable tool for all our verifications.

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