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Optics of Gaussian Beams and Digital Resonator Using MATLAB

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Abstract-Our research focuses on the theoretical programs used to discuss the background and theory behind the optics of Gaussian beams and the implementation of digital resonances using MATLAB.

In the scope of this paper, we will see why we are concerned about the mode size and location of the resonant, and the advantages and disadvantages of stable and unstable resonator. When discussing laser beam parameters such as mode waist and divergence in this paper, the laser is supposed to operate in TEM_{00} mode. The TEM_{00} process is particularly important for effective diode pumping. The resonator mode waist is the smallest of all cross-sectional conditions or transverse modes, thus providing the opportunity for excellent matching of the pump beam, which is highly concentrated in the geometry of the end-pumped. The size mode volume is also small, so that, for the either side pumping or the end pumping, the flow of the diode pump can be concentrated within the laser resonator mode to produce high pump power density. This allows the development of high gain within the resonator, and produces high pump efficiencies for the output from optical to laser.

Keywords- *Gaussian Beams, Modes, Resonator Stability*

I. INTRODUCTION

Optical resonance, the optical resonance part of the electronic resonance circuit, stores and stores light at certain resonance frequencies, and can be viewed as an optical transmission system that includes feedback; light is reflected or reflected repeatedly within the system, without escaping. The simplest resonance includes two parallel planar (flat) mirrors between which light is reflected repeatedly with little loss. Typical optical resonator configurations are described in Fig. l [1].

The frequency selectivity of optical resonance makes it useful as a light filter or spectrum analyzer. However, its most important use is the "container" through which the laser light is generated. A laser is a light resonator that contains a medium that amplifies light. The frequency modulator determines the spatial distribution of the laser beam. Because resonances have the capability to storing energy, they can also be used to generate laser energy pulses [1].

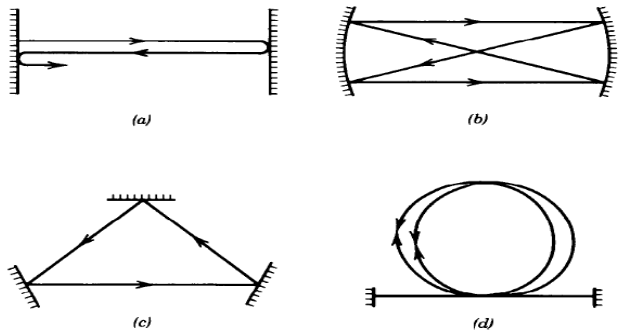

Figure 1. Optical resonators: (a) planar-mirror resonator; (b) spherical-mirror resonator; (c) ring resonator; (d) optical-fiber resonator [1].

II. THEORY

The Gaussian TEM_{00} beam can be distinguished on the beam waist by its amplitude, u_0, and the beam size (defined as the 1/e radius), w_0. The beam is smallest at the beam waist is sized and the radius of the wavefront is infinitive, meaning it contains it has a plane wavefront. In formula [2]:

$$u(r) = u_0 e^{-(\frac{r}{w_0})^2} \quad (1)$$

The intensity of the beam is proportional to the square of the amplitude and can be written as [2]:

$$I(r) = I_0 e^{-2(\frac{r}{w_0})^2} \quad (2)$$

When a Gaussian beam propagates in the z direction from its waist at $z = 0$ to a point z, it can be shown that the radius of its wavefront is given by [2]:

$$R(z) = z + \frac{z_R^2}{z} \quad (3)$$

Where z_R is the Rayleigh range [2], in a resonator with two mirrors positioned at $z=z_1$ and $z=z_2$ respectively, the radii of the wavefronts at the mirrors must be equal to the radii of curvatures, R_1 and R_2 respectively, of the mirrors (Fig. 2). The size of the beam inside the resonator can thus be found by solving [2]:

$$R(z_1) = z_1 + \frac{z_R^2}{z_1} = -R_1 \tag{4}$$

$$R(z_2) = z_2 + \frac{z_R^2}{z_2} = R_2 \tag{5}$$

$$L = z_2 - z_1 \tag{6}$$

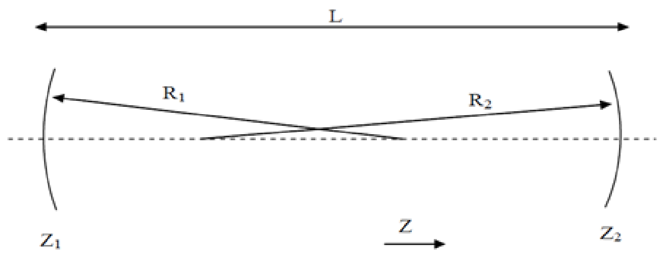

Figure 2. Resonator with two mirrors [1].

The minus sign in the first equation arises because of the sign convention used for mirror- and wavefront radii. We define the so-called g-parameters of the resonator [2]:

$$g_1 = 1 - \frac{L}{R_1} \tag{7}$$

$$g_2 = 1 - \frac{L}{R_2} \tag{8}$$

It can be found by solving (1) and using (2), that [2]:

$$z_R^2 = \frac{g_1 g_2 (1 - g_1 g_2)}{(g_1 + g_2 - 2 g_1 g_2)^2} L^2 . z_1 = \frac{g_2 (1 - g_1)}{g_1 + g_2 - 2 g_1 g_2} L . z_2 = \frac{g_1 (1 - g_2)}{g_1 + g_2 - 2 g_1 g_2} L \tag{9}$$

$$w_0^2 = \frac{L\lambda}{\pi} \sqrt{\frac{g_1 g_2 (1 - g_1 g_2)}{(g_1 + g_2 - 2 g_1 g_2)^2}} = \frac{\lambda z_R}{\pi} \tag{10}$$

$$w_1^2 = \frac{L\lambda}{\pi} \sqrt{\frac{g_2}{g_1 (1 - g_1 g_2)}} \tag{11}$$

$$w_2^2 = \frac{L\lambda}{\pi} \sqrt{\frac{g_1}{g_2 (1 - g_1 g_2)}} \tag{12}$$

Where λ is the laser wavelength, w_1 and w_2 are the beam sizes at the mirrors, L, is the distance between the mirrors, and w_0 is the beam waist at $z = 0$. From the equations it can be seen that real solutions only exist if [2]:

$$0 \leq g_1 g_2 \leq 1 \tag{13}$$

This is called the stability range of the resonator. Outside this range no beam can exist between the mirrors and hence no radiation will be emitted by the laser.

III. Resonator Stability

Optical resonance is either stable or unstable with respect to the transverse beam compensation. If it is stable, any geometrical beam is injected into the system with some positions and the initial non-large transverse angle will remain in the system during many circular trips. In an unstable resonator, this beam will be ejected sooner or later. The characteristics of the resonator modes vary considerably in the stable or unstable system. Unstable resonances contain a number of special properties [3]:

1. These patterns are always experience a large loss of the diffraction, which is mostly very high (at a rate of 50% per round trip or higher).
2. The diffraction losses generally become higher for higher mode orders. This fundamental distinction can help one obtain a single laser transverse process.
3. Particularly for the hard-edged diffraction resonators, the transverse mode profiles of the lateral position are very complex and usually show clear ring structures.

In the unstable linear resonator, the wave fronts of the counter-propaganda beams do not necessarily correspond to each other, and do not necessarily correspond to the surfaces of the mirrors of the two end mirrors [3].

IV. Resonator Modes

Gaussian modes are the lower order patterns modes of an optical resonator, along the transverse trend. The stable resonator will have homogeneous optical media; a flat surface band or parabolic between the media. Therefore, laser beams in the primary transverse mode are often emitted close to the Gaussian shape.

On the other hand, high transverse orders are known as Laguerre-Gaussian or Hermite-Gaussian functions. Any deviation from the Gaussian beam shape can be estimated by M^2 factor. The Gaussian beam, which has the highest quality beam, matches with $M^2 = 1$ [4, 5].

V. Results And Discussion

In this paper, we study the results of MATLAB Gaussian Beam Model; this calculates the normalized pressure at normal incidence through a spherically curved interface:

1. Input parameters.
2. Used the coefficients of Wen and Breazeale to calculate the wave field. A powerful Gaussian-beam expansion technique [6] is widely used in calculating the Fresnel field integral.
3. Calculates the beam size inside the resonator for given laser wavelength, mirror radii and resonator length.

4. Calculate the transmission coefficient, interface curvature and the Gaussian beam model, then plot magnitude on-axis.

Fig. 3 shows the relationship between position of mirrors and beam size at mirrors. Fig. 4 shows the relationship between resonator g-parameters g_1 and g_2. We define the so-called g-parameters of the resonator in 7 and 8.

Mathematically model beam propagation of Gaussian beam using simple geometric parameters, the software uses first order approximations and assumes TEM_{00} mode to determine beam spot size in free space applications.

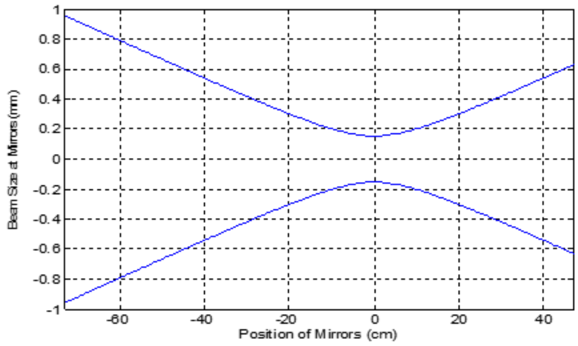

Figure 3. TEM_{00} beam size inside the resonator. The relationship between position of mirrors and beam size at mirrors.

The width of the TEM_{00} Gaussian mode inside the resonator is plotted as well as the stability diagram, where the two magenta curves are for resonator g-parameters g1g2= 1.The first mirror is on the left side and the second mirror on the right side. The blue dot in the stability diagram refers the resonator. The blue dot should be inside the area specified by the magenta curves and g_1= 0, g_2= 0 lines. If it is outside this area, the resonator is not confined and no mode can exist.

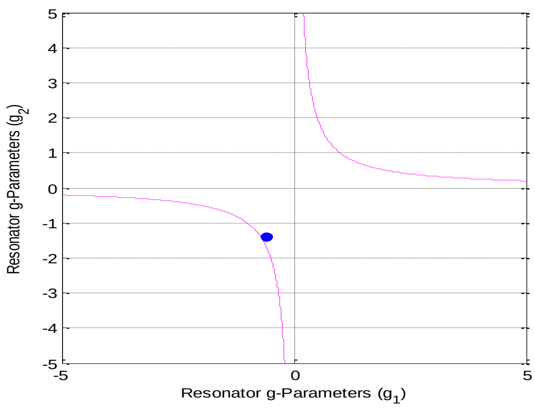

Figure 4. Stability diagram. The relationship between resonator g-parameters g_1g_2.

Figs. (5 – 7) shows the relationship between path length and pressure, frequency = (5, 10 and 15) MHz.

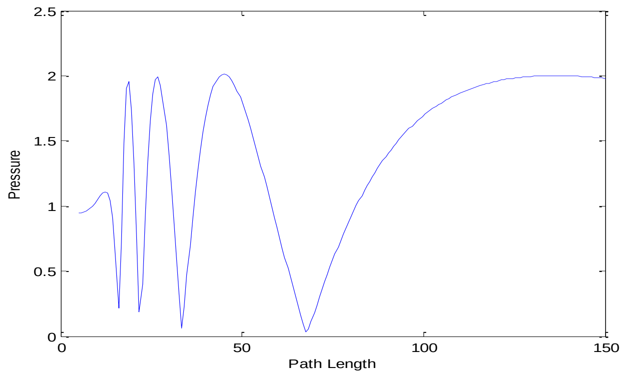

Figure 5. The relationship between path length and pressure, frequency = 5 MHz.

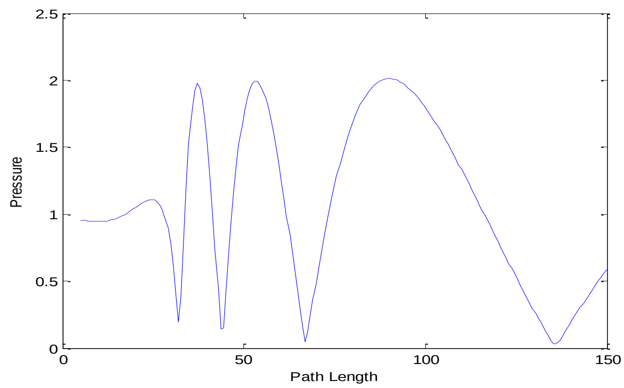

Figure 6. The relationship between path length and pressure, frequency = 10 MHz.

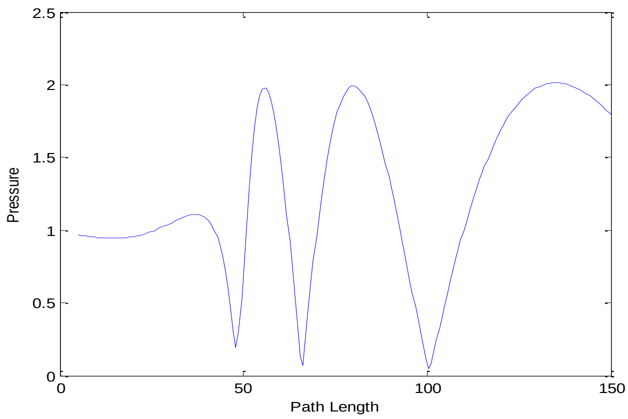

Figure 7. The relationship between path length and pressure, frequency = 15 MHz.

Figs. (8 – 10) shows the relationship between distance from ray axis and pressure, frequency = (5, 10 and 15) MHz. The cross-axis plot at path length= 130 mm (near last maximum. The transducer radius= 6.35 mm.

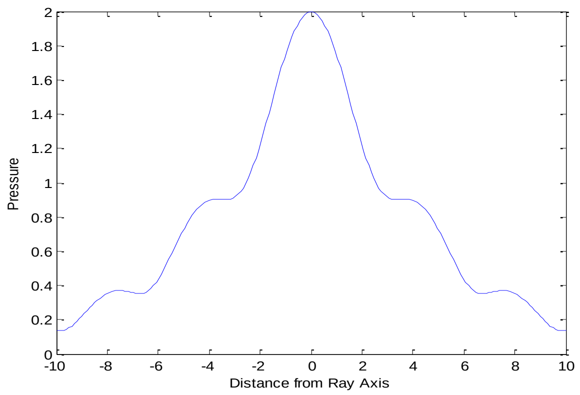

Figure 8. The relationship between distance from ray axis and pressure, frequency = 5 MHz.

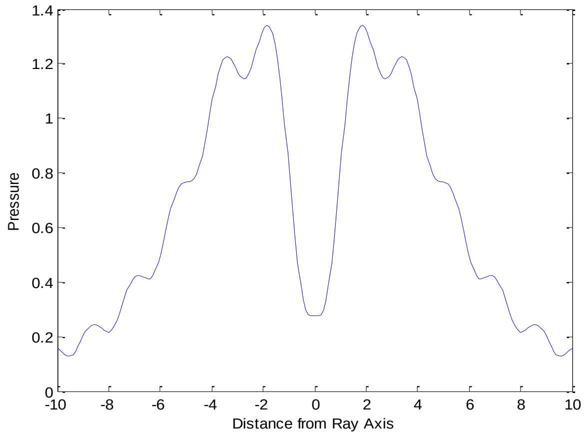

Figure 9. The relationship between distance from ray axis and pressure, frequency = 10 MHz.

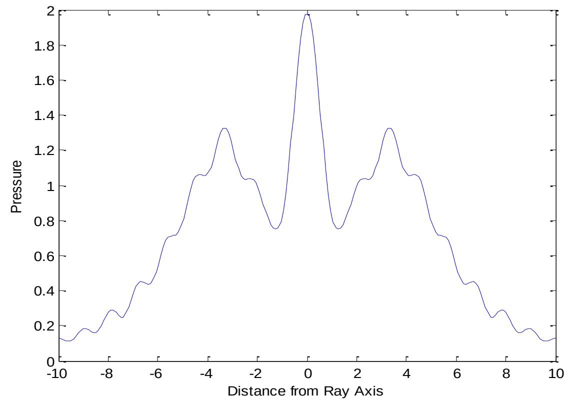

Figure 10. The relationship between distance from ray axis and pressure, frequency = 15 MHz.

Figures 11–13 show the relationship between distance from ray axis and pressure, frequency = (5, 10 and 15) MHz. The cross-axis plot at path length= 65 mm (near last on-axis null). The transducer radius= 6.35 mm.

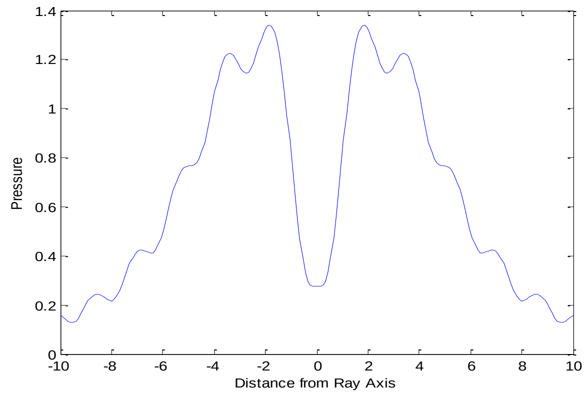

Figure 11. The relationship between distance from ray axis and pressure, frequency = 5 MHz.

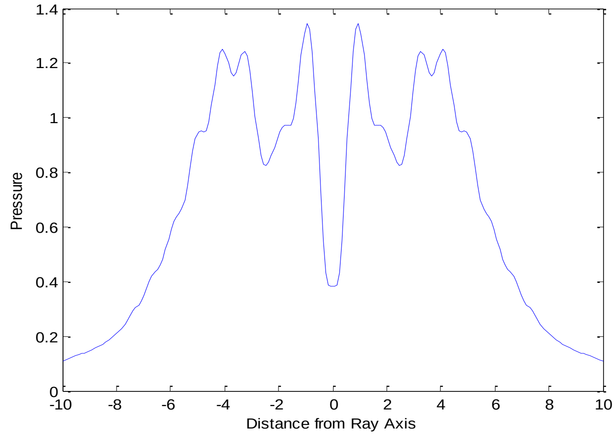

Figure 12. The relationship between distance from ray axis and pressure, frequency = 10 MHz.

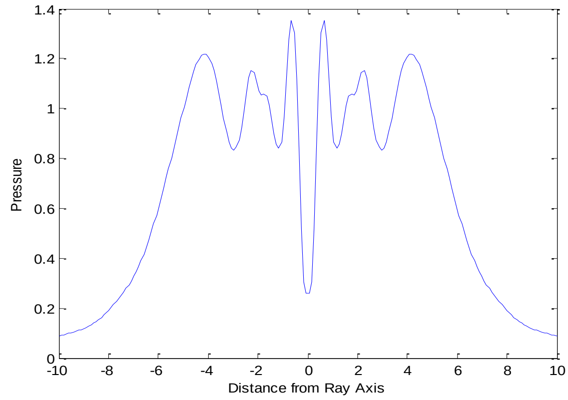

Figure 13. The relationship between distance from ray axis and pressure, frequency = 15 MHz.

VI. CONCLUSIONS

Resonators can be stable or unstable. Stability is determined by the radius of the curvature of the mirrors, the spacing between the mirrors, and the refractive index of the material in the recycled circulation path. In a stable resonator, the lower order conditions remain close to the optical axis, the diffraction loss is small and the stimulated emission occurs only within a relatively slender small size in the gain element. Unstable resonances, on the other hand, have mode volumes that are usually much larger. These resonances are characterized by significant loss of diffraction. In fact, the diffraction diffuse through one of the end mirrors can be used as the output coupling mechanism.

TEM00 mode is the lower-case mode. It has a lower threshold, smaller beam waist and divergence, and has no nodes in the intensity distribution of output beam transverse.

There are several resonator mode parameters useful for discussing in general. One important parameter is the mode waist position, which indicates the minimum cross-sectional radius of the transverse beam inside the laser resonator. The cross-beam beam radiates in the mirrors of both sides, the waist position for each mirror and the separation of mirrors.

TEM_{00} mode has many features that make it a desirable design goal. The beam divergence is low, providing high power density at large propagation distances and because the output beam intensity has a uniform Gaussian profile, it is useful for many lighting illumination and imaging applications.

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