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Influence of the Double-Cone Starting Position, on the Swinging Divergent-Convergent Straight Rails, upon Its Travelling Period

Barenten Suciu[1], Yuya Ohmura[2], Kosuke Takeda[3]
[1,2,3]Department of Intelligent Mechanical Engineering, Fukuoka Institute of Technology, Fukuoka, 811-0295 Japan
([1]suciu@fit.ac.jp, [2]s15e2016@bene.fit.ac.jp, [3]s15e2041@bene.fit.ac.jp)

Abstract-In this paper, the influence of the double-cone starting position, on swinging divergent-convergent straight rails, upon its travelling period is investigated. Concretely, rails are fixed on the hull of a model ship, which is excited to roll along its longitudinal axis, by using a pendulum. Movies of the double-cone rolling on the rails were shot, and the total travelling period was determined through the slow motion processing of the films, for various start positions of the double-cone on the rails, and values of the maximal inclination angle of the ship hull, i.e., several potential energies initially introduced into the system. In order to explain the experimental findings, and the significance of the observed critical inclination angle of rails, a theoretical model is advanced. Results obtained are discussed from an applicative standpoint.

Keywords- *Double-Cone, Divergent-Convergent Straigth Rails, Starting Position, Pendulum, Dynamics*

I. INTRODUCTION

Recently, wave-powered electrical generators, employing double-cones moving on circular rails, have been proposed [1]. Rotational motion of the generator's hull, induced by waves, is changed into the rotational and translational movement of either a rigid magnetized double-cone, rolling on divergent-convergent circular rails [1], or a double-cone geared motor-generator, rolling on concentric circular rails [2]. Thus, either a rigid double-cone composed of two fixedly joined cones [3]-[8], or a double-cone where the relative rotation motion between the inner and outer cones is permitted by a rotational-link, i.e. by a geared motor-generator [2], can be employed.

However, if such power generator is not directly floating on the water waves of random direction, period and height, but it is placed inside the hull of a ship, movement of the double-cone becomes one-directional and it is dictated by the rolling period and amplitude of the ship's hull. In such case, divergent-convergent straight rails, resembling an O letter, might be more efficient than the circular rails. Previously, the rails optimal geometry was decided in order to maximize the kinetic energy of translation of the double-cone [9]. Then, the apparent spring constant and the damping ratio, for the oscillatory movement of the double-cone, were experimentally found and validated by a theoretical model [9]. Experimentally observed optimal length of the pendulum arm, to maximize the total travelling time of the double-cone, was justified by a two degree of freedom vibration model of the double-cone/pendulum mechanism [9].

In this work, the effect of the double-cone starting position, on the tilting divergent-convergent straight rails, upon its total travelling time is examined. For this reason, rails are fixed on the hull of a reduced-scale ship, which is excited to roll along its longitudinal axis, by using a pendulum. Films of the double-cone rolling on the rails are shot, and the total travelling time is determined through the slow motion processing of the movies, for various values of the inclination angle of the hull, and start positions of the double-cone on the rails, i.e., different potential energies initially given to the system. Influence of the maximal inclination angle of the rails, in correlation with the starting position of the cone, as well as the significance of the critical inclination angle of the rails, is clarified.

II. TEST RIG AND EXPERIMENTAL PROCEDURE

Fig. 1 illustrates the test rig, which is mainly consisted of a ship hull, a double-cone mechanism, a pendulum of mass and length a stopper, and a protractor. Double-cone of mass is moving on a set of divergent-convergent rails fixed on the ship hull. Actual ship, excited by the water waves, is rolling along its longitudinal axis. However, in order to excite the rolling vibration of the reduced-scale ship from Fig. 1, initial potential energy is given to pendulum. Maximal angle is set to the desired value of 1, 2, 3, 4, 5, 6, 7, and 8 deg, by adjusting the position of the stopper. Instantaneous angle is found by using both a traditional protractor and a protractor-like application on a smartphone, attached to the ship hull. Pendulum's oscillation period can be controlled by adjusting the length in the range from 50 to 600 mm, to achieve synchronization conditions [9]. Change of potential energy introduced into the system due to the modification of the maximal angle or the length can be kept under control by varying the mass in the range of 0.5 to 5 kg. Hull's rolling motion excites the reciprocating movement of the double-cone on the rails.

Concerning the reduced-scale ship hull, it has a total length of 815 mm, a maximal width of 220 mm, and a maximal height of 237 mm. Ship hull is fixed to a shaft of 900 mm length, which is supported at both ends by bearing units attached to pillar supports. Divergent-convergent rails unit is fixed at the middle-upper part of the hull. In order to prevent the derail of the double-cone, rail stoppers are provided at the both ends of the divergent-convergent rails (see Figs. 1 and 2).

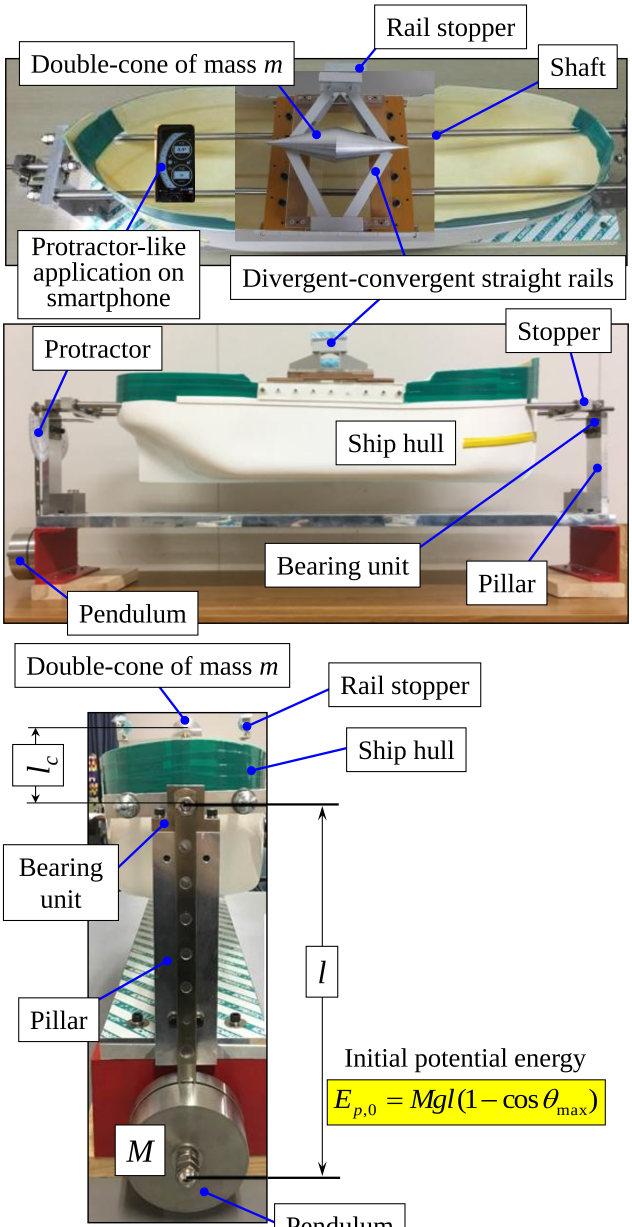

Figure 1. Photograph of the test rig, mainly consisted of a ship hull, a double-cone mechanism, a pendulum, a stopper, and a protractor device.

Two cones, made in A5052 aluminum alloy, each having a height of $H = 98.55$ mm, a radius at the base circle of $R = 25$ mm, and a radius at the apex circle of $R^* = 0.35$ mm, are joined together, by using a bonding adhesive, to achieve the so-called double-cone. Thus, the apex angle of the double-cone is of $\Psi = \tan^{-1}[(R - R^*)/H] = 14.043$ deg. Mass of the double-cone is of $m = 0.358$ kg, and the mass inertia moment, around the double-cone's longitudinal axis of symmetry, is of $I = 0.3mR^2[1-(R^*/R)^5]/[1-(R^*/R)^3] = 67$ kg·m². Rails are made in the same A5052 aluminum alloy, with a modulus of elasticity of $E = 74$ GPa, and a Poisson ratio of $\nu = 0.33$.

To achieve the maximal kinetic energy of translation for the double-cone [7], [8], the optimal geometry of the rails was decided, for the given width of the hull [9]. Thus, Fig. 2 shows the geometry of the test rig rails, adopted as follows: entrance span, $L_2 = 0$ mm; effective entrance span, 13 mm; exit span, $L_1 = 122$ mm; rail length, $L_0 = 119$ mm; effective rail length, 106 mm; thickness of the rails stopper, $h = 11$ mm; rails total span, 204 mm; rails effective span, 182 mm; rails opening angle, $\Phi = \sin^{-1}(0.5L_1/L_0) =$ 30.838 deg; cone-rails contact angle, $\alpha = \sin^{-1}(\tan\Phi \cdot \tan\Psi) = 8.588$ deg; start position of the minimal length, $L_S = (h + R)/\cos\Phi =$ 42 mm, that corresponds to the minimal start position $x_{0,\min} = -66$ mm on Fig. 3.

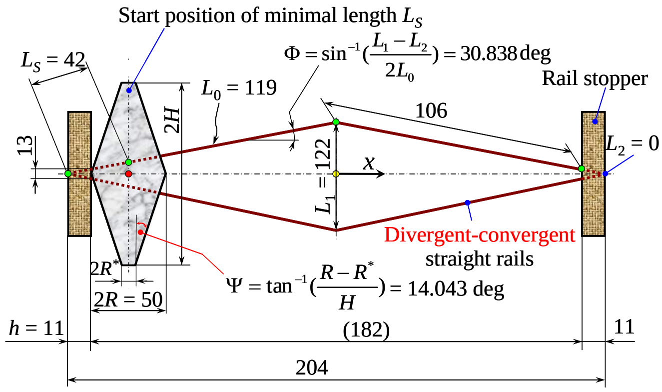

Figure 2. Adopted geometry for the rails and the double-cone used in the forced vibration tests under pendular excitation.

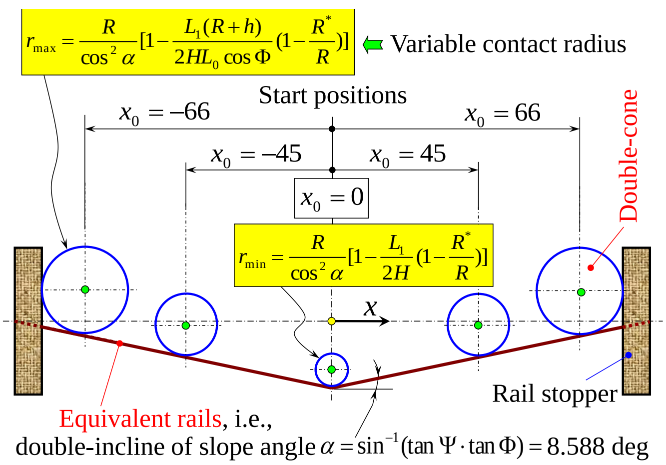

Figure 3. Equivalent double-cone mechanism, occuring as an equivalent cylinder of variable contact radius, rolling on equivalent rails, i.e., on a double-incline of slope equal to the contact angle.

Although the divergent-convergent rails used in this work are planar (see Figs. 1 and 2), due to the particular contact geometry, the double-cone mechanism can be regarded as a cylinder of variable contact radius rolling on a double-incline with a slope, i.e., a contact angle of $\alpha = 8.588$ deg (see Fig 3).

During experiments five start positions of the double-cone on the rails were considered, corresponding to the following values of the coordinate $x_0 = -66;-45;0;+45;+66$ mm (see Fig. 2 and 3). As illustrated in the results section, the start position of the double-cone on the rails has considerable influence on the total travelling time. On calm sea water of negligible waves, the energy introduced into the mechanical system from the waves is scarce. Accordingly, the double-cone is likely to start its movement from the central position of $x_0 = 0$ mm that corresponds to the most stable state, of minimal potential energy. However, on agitated sea water, for the usual gravity waves with a period of 8-13 s [1], [2], the total rolling time of the double-cone is longer than the waves period (see Figs. 5 and 6). Therefore, the double-cone is likely to "restart" its movement at locations deviated from the central position.

Films of the double-cone moving on the rails were shot, and the total travelling time T_t was determined through the slow motion processing of the movies, and also by using a stopwatch, for various values of the maximal inclination angle of the hull, and start positions of the double-cone on the rails, i.e., several potential energies initially given to the system. Reciprocating motion of the double-cone, considered as a material point, can be perceived as an oscillatory translation motion of variable period $T_{tr,j}$; $j = \overline{1-n}$, where n is number of cycles until the double-cone arrives to stop at the central position $x = 0$. On the other hand, by considering the double-cone as a body of revolution, the number of rotations $N_{i,j}$ and the variable period of rotation $T_{rot,i,j}$ can be found from the slow motion analysis of the movies. For one cycle j of reciprocating motion, the period of translation can be calculated as $T_{tr,j} = \sum_{i=1}^{N_{i,j}} T_{rot,i,j}$, and then, the total travelling time as $T_t = \sum_{j=1}^{n} T_{tr,j}$. Based on such relationships between the temporal parameters, a thorough verification for correctness of the partial and final experimental results can be done.

III. Experimental Results for Forced Vibration of the Double-Cone under Pendular Excitation

Fig. 4 shows snapshots of the double-cone, forcedly rolling on the divergent-convergent straight rails attached to the ship hull, for different values of the maximal inclination angle $\theta_{\max} = 1$, 2, 3, 4, 5, 6, 7, 8 deg, and five start positions, of coordinates $x_0 = -66;-45;0;+45;+66$ mm.

Note that the pendulum start position is set to $-\theta_{\max}$, i.e., toward the left side in the lower part of Fig. 1. In this case, for a start position of the double-cone on the rails at $x_0 = -66$ mm, the initial phase angle between pendulum and cone is nil, i.e., initially, they oscillate in the same phase. This appears as quite beneficial for augmenting the amount of energy transferred from the rails to the double-cone (Figs. 5-6). On the other hand, for a start position of the double-cone on the rails at $x_0 = 66$ mm, the initial phase angle between the pendulum and cone is of 180 deg, i.e., initially, they oscillate in the opposition of phase. In such case, the transfer of energy from the rails to the double-cone occurs as being less effective (Figs. 5-6). Motion of the double-cone for $x_0 = 0$ and small values of the maximal inclination angle $\theta_{\max}$ cannot be achieved, since the initially inputted potential energy into the system is insufficient.

Figure 4. Snapshots of the double-cone, forcedly rolling on the divergent-convergent straight rails fixed at the ship hull, for different values of the maximal inclination angle and start coordinate.

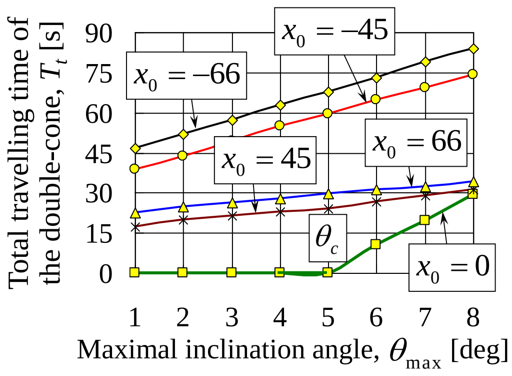

Figure 5. Variation of the total travelling time of the double-cone versus the maximal inclination angle, for various values of the start coordinate.

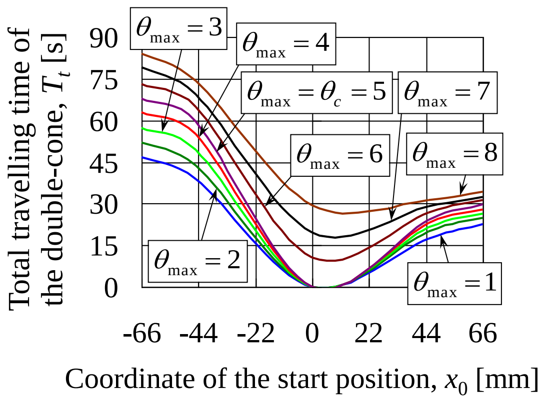

Figure 6. Variation of the total travelling time of the double-cone versus the start coordinate, found for various values of the maximal inclination angle.

IV. INTERPRETATION OF THE CRITICAL INCLINATION ANGLE OF THE RAILS

In order to explain the significance of the critical inclination angle θ_c (see Fig. 5), i.e., the value of the maximal inclination angle of the rails from which the movement of the double-cone becomes possible for $x_0 = 0$, a theoretical model is suggested, as follows. Thus, one firstly takes into account the equilibrium of forces acting on the double-cone, when it is stopped at the position of the potential well $x_0 = 0$, for a certain inclination angle θ of the pendulum (see Fig. 7), as follows:

$$\begin{cases} (N_1 + N_2)\cos\alpha + ml_c\dot{\theta}^2 - mg\cos\theta = 0 \\ (N_1 - N_2)\sin\alpha + mg\sin\theta + ml_c\omega_p^2\theta = 0 \end{cases} \qquad (1)$$

Here N_1 and N_2 are the normal forces at the contact of the double-cone with the left and right rails, respectively. On the other hand, mg is the weight of the double-cone, $ml_c\dot{\theta}^2$ is the centrifugal force, and $-ml_c\ddot{\theta} = ml_c\omega_p^2\theta$ is the inertia force. Assuming an ideal dynamics [9] for the test pendulum of Fig. 1, the pendular inclination angle θ can be written as:

$$\theta = -\theta_{max}\cos\omega_p t \quad ; \quad \omega_p = \sqrt{g/l} \qquad (2)$$

In such conditions, the corresponding angular velocity $\dot{\theta}$ and acceleration $\ddot{\theta}$ can be calculated as:

$$\dot{\theta} = \omega_p\theta_{max}\sin\omega_p t \quad ; \quad \ddot{\theta} = \omega_p^2\theta_{max}\cos\omega_p t = -\omega_p^2\theta \qquad (3)$$

In the above equations, $l_c = 60$ mm is the distance from the pendulum center of rotation to the mass center of the double-cone, and $l = 280$ mm is the length of the pendulum arm (see Figs. 1 and 7).

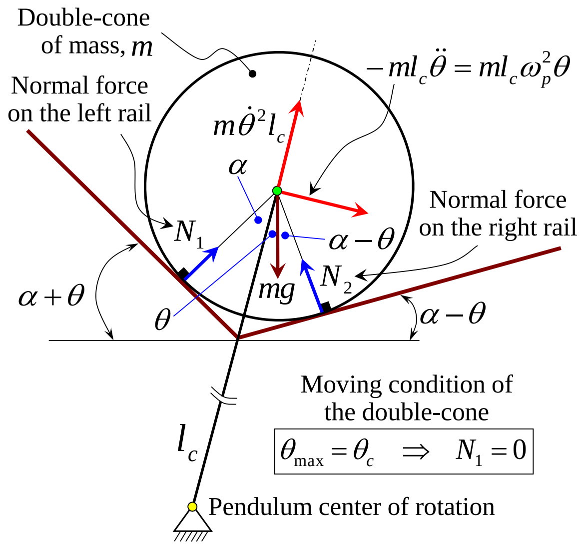

Figure 7. Mechanical model for the critical inclination angle, showing the equilibrium of forces acting on the double-cone, when it is stopped at the position of the potential well.

For small values of the contact angle $\alpha = 8.588$ deg and maximal inclination angles ($|\theta_{max}| \le 8$ deg), it is reasonably to consider that $\cos\alpha \cong 1$, $\cos\theta \cong 1$, $\sin\alpha \cong \alpha$, and $\sin\theta \cong \theta$. In these circumstances, linearization of the set of equations (1) can be achieved, as follows:

$$\begin{cases} (N_1 + N_2) + mg\dfrac{l_c}{l}\theta_{max}^2\sin^2\omega_p t - mg = 0 \\ (N_1 - N_2)\alpha + mg\theta + mg\dfrac{l_c}{l}\theta = 0 \end{cases} \qquad (4)$$

Then, from the lower equation (4), one finds the normal force N_2 as:

$$N_2 = N_1 + mg\frac{\theta}{\alpha}(1 + \frac{l_c}{l}) \ge N_1 \qquad (5)$$

which as expected, always exceeds the normal force N_1. Note that, the state of repose or movement for the double-cone can be determined by analyzing the sign of the normal force N_1 (see Fig. 7). With this purpose, by substituting (5) in the upper equation (4), one obtains the normal force N_1 as:

$$N_1 = \frac{mg}{2}[1 - \frac{\theta}{\alpha}(1 + \frac{l_c}{l}) - \frac{l_c}{l}\theta_{\max}^2 \sin^2 \omega_p t] \tag{6}$$

When the double-cone starts to move, the contact with the left rail is no longer maintained. Such condition can be written from a mathematical standpoint as $N_1 \le 0$, which based on (6), leads to the following inequality to be satisfied:

$$\frac{\theta}{\alpha}(1 + \frac{l_c}{l}) + \frac{l_c}{l}\theta_{\max}^2 \sin^2 \omega_p t \ge 1 \tag{7}$$

Evidently, the condition (7) might be not fulfilled for any instant of time. However, it is sufficient to be fulfilled for the most convenient instant of time, which can be decided as follows. Since the contact angle α is constant and $\theta_{\max} < \alpha$, one concludes that the most favorable instant of time to enable the motion of the double-cone is $t = \pi / \omega_p$, for which the inclination angle becomes $\theta = \theta_{\max}$, and therefore, the right-rail from Fig. 7 reaches the minimal value $\alpha - \theta_{\max}$ of its slope. In these circumstances: $\sin^2 \omega_p t = 0$, and the inequality (7) reduces to (8), which in turn, provides the expression for the critical inclination angle of the rails:

$$\theta_{\max} \ge \theta_c = \frac{\alpha}{1 + l_c / l} \tag{8}$$

One observes that, substituting the values above mentioned for the parameters used in this experimental work, the critical inclination angle can be found as: $\theta_c = 8.588$ deg / (1 + 60 mm/ 280 mm) $\cong$ 7 deg. Under the limitations of the proposed linear model and ideal dynamics of the pendulum, one concludes that the theoretically obtained critical angle of 7 deg reasonably agrees with the experimentally determined value of $\theta_c \cong 5$ deg (see Fig. 5).

V. Potential Energy of the Double-Cone at the Starting Position

In order to explain the results shown by Figs. 5 and 6, one take into account the magnitude of the double-cone potential energy at the starting position on the rails (see Fig. 8).

With this end, one firstly clarifies the contact radius of the double-cone with the rails, corresponding to different starting positions. Thus, when the double-cone is placed at the position of the potential well $x_0 = 0$, the contact radius reaches its minimal value $r_{\min} =$ 9.964 mm $\cong$ 10 mm, given by [7], [9]:

$$\begin{aligned} r_{\min} &= \frac{R}{\cos^2 \alpha}[1 - \frac{L_1}{2H}(1 - \frac{R^*}{R})] = \\ &\frac{25mm}{\cos^2 8.588^0}[1 - \frac{122}{197.1}(1 - \frac{0.35}{25})] \cong 10mm \end{aligned} \tag{9}$$

On the other hand, at the rails extremities $x_{0,e} = \pm 66$ mm, the contact radius reaches its maximal value $r_{\max} =$ 20.062 mm $\cong$ 20 mm, given by [7], [9]:

$$\begin{aligned} r_{\max} &= \frac{R}{\cos^2 \alpha}[1 - \frac{L_1(R+h)}{2HL_0 \cos \Phi}(1 - \frac{R^*}{R})] = \\ &\frac{25mm}{\cos^2 8.588^0}[1 - \frac{122 \times 42}{197.1 \times 119}(1 - \frac{0.35}{25})] \cong 20mm \end{aligned} \tag{10}$$

Finally, at the intermediate start positions $x_{0,i} = \pm 45$ mm, the contact radius reaches its intermediate value $r_i =$ 16.818 mm $\cong$ 16.8 mm, given by [7], [9]:

$$\begin{aligned} r_i &= r_{\min} + (r_{\max} - r_{\min}) \left| \frac{x_{0,i}}{x_{0,e}} \right| = \\ &= 10mm + 10mm \left| \frac{\pm 45}{\pm 66} \right| \cong 16.8mm \end{aligned} \tag{11}$$

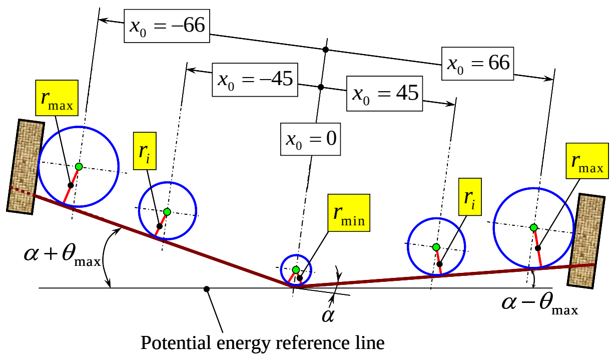

Figure 8. Geometrical model of the inclined rails used to evaluate the initial potential energy of the double-cone at various starting positions on the rails.

Next, based on the geometrical model of the rails shown by Fig. 8, one expresses the initial potential energy $E_{p,dc,0}$ of the double-cone corresponding to various starting positions, as follows. When the double-cone is placed at the position of the potential well $x_0 = 0$, the initial potential energy $E_{p,dc,0}$ of the double-cone is given by:

$$E_{p,dc,0} = mgr_{\min} \frac{\cos \theta_{\max}}{\cos \alpha} \tag{12}$$

On the other hand, at the rails extremities $x_{0,e} = \pm 66$ mm, the initial potential energy $E_{p,dc,0}$ of the double-cone can be written as:

$$E_{p,dc,0} = mg[|x_{0,e}|\sin(\alpha \pm \theta_{\max}) + r_{\max}\cos(\alpha \pm \theta_{\max}) \quad (13)$$

where the sign plus is corresponding to $x_{0,e} = -66$ mm, and the sign minus is related to $x_{0,e} = 66$ mm. At the intermediate start positions $x_{0,i} = \pm 45$ mm, the initial potential energy $E_{p,dc,0}$ of the double-cone can be calculated as:

$$E_{p,dc,0} = mg[|x_{0,i}|\sin(\alpha \pm \theta_{\max}) + r_i\cos(\alpha \pm \theta_{\max}) \quad (14)$$

where again, the sign plus is corresponding to $x_{0,i} = -45$ mm, and the sign minus is related to $x_{0,i} = 45$ mm.

Fig. 9 shows, based on (9-14), the variation of the initial potential energy $E_{p,dc,0}$ of the double-cone versus the maximal inclination angle $\theta_{\max} = 1, 2, 3, 4, 5, 6, 7, 8$ deg, obtained for five start positions: $x_0 = -66;-45;0;+45;+66$ mm. One notes that the initial potential energy increases for $x_0 = -66$ mm and $x_0 = -45$ mm, remains almost unchanged for $x_0 = 0$, and decreases for $x_0 = 45$ mm as well as $x_0 = 66$ mm against the maximal inclination angle. Based on the resemblance between Figs. 5 and 9, relative to the variation versus the initial position, one suggests that the total travelling time of the double-cone is mainly dictated by the magnitude of its initial potential energy.

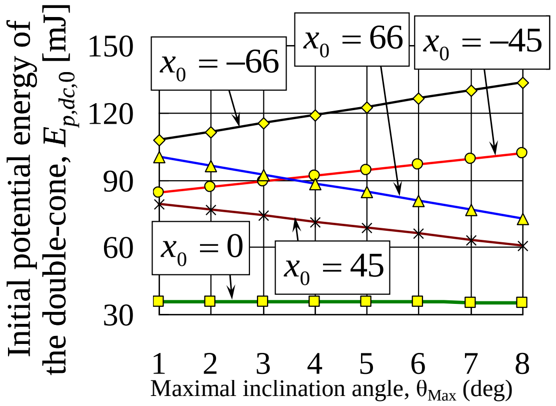

Figure 9. Variation of the initial potential energy of the double-cone versus the maximal inclination angle of the rails, obtained for different starting positions of the double-cone on the rails.

VI. SUMMARY

In this work, the influence of the double-cone starting position, on tilting divergent-convergent straight rails, upon its travelling period was experimentally investigated, and then, was theoretically argued. During forced rolling tests, linear augmentation of the total travelling time was noticed against the maximal inclination angle of the hull. Moreover, the nonlinear influence of the start coordinate on the total travelling time was emphasized, and its significance, relative to the period of the gravity water waves, was discussed. In the end, significance of the critical inclination angle, associated to the potential well, was taken into consideration, and a simple mechanical model, to evaluate this angle, was advanced. Experimental and theoretical evidence suggests that the total travelling time of the double-cone is mainly dictated by the magnitude of its initial potential energy.

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