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Dynamic Analysis of a Class of Impact Vibration System with Clearance

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Abstract- In this paper, the dynamic analysis of a kind of impact vibration system with clearance is carried out. A theoretical model of six-dimensional three-degree-of-freedom collision system is established according to engineering requirements. The collision equation is analyzed by modal analysis method, and the periodic solution of the system is obtained. The stability problem of the system is transformed into the discussion of Poincaré mapping by using Poincaré mapping. By finding Jacobi matrix of Poincaré map at the fixed point, the critical parameters of bifurcation are obtained. Using numerical simulations, the characteristic curves of period doubling bifurcation of the system under certain parameters are drawn, and the period doubling bifurcation points of the system are found.

Keywords- *Modal Analysis, Poincaré Mapping, Jacobi Matrix, Period Doubling Bifurcation*

I. INTRODUCTION

Nowadays, the non-linear impact vibration system with clearance is widely used in practical engineering field, such as impact centrifugal dehydrator, vibrating sand blasting machine, vibrating roller, pile driver and so on. However, in the other part, the repeated impact vibration of parts contacting, detaching, re-contacting and re-detaching may lead to the damage of machine parts, for example, in nuclear reactor, the repeated collision of fuel rod and reactor shell may lead to the wear and even damage of heat exchanger tubes.

In addition, impact vibration can also cause environmental noise pollution, such as the working environment of forging factories. In the high-speed operation of trains, the collision between wheels and tracks will not only affect passengers' riding experience, but also threaten the safety of the train. Therefore, the research and control of the non-linear impact vibration system have theoretical significance and practical value.

At present, in the field of vibration system with clearance, many scholars at home and abroad have carried out exploratory research. Impact collision will lead to discontinuity and strong nonlinearity of vibration system with clearance, which makes its dynamic characteristics complex and changeable. Li and Tian [1] established a kind of single-degree-of-freedom elastic collision system based on the rear axle leaf spring system of vehicles. The Poincaré mapping method was used to analyze the system. It was proved that there were periodic doubling bifurcations and almost periodic bifurcations in the vehicle leaf spring system, and the evolution process of periodic motion to chaotic motion was presented. Zhu and Shen [2] took a piecewise linear system as the research object, and found bifurcation and chaos phenomena through related calculation. The cell transformation method is used to analyze some system parameters comprehensively. The attraction domain of the attractor is drawn and the existence of Smale horseshoe is proved. Zhao [3] established a two-degree-of-freedom bilateral collision system under harmonic excitation, deduced the parameters of analytical solution and its existence conditions, and analyzed its response. Shaw [4] discussed the chaos and long-period motion of a class of linear oscillators with single degree of freedom. Li [5] analyzed two single-degree-of-freedom relative impact vibration systems and obtained the existence criterion of subharmonic motion and the stability condition of periodic motion using Jacobi matrix eigenvalue method. Foale [6] found that the discontinuity of vibro-impact system can lead to bifurcation, which cannot be classified by common bifurcation analysis theory. Luo [7] took the two-degree-of-freedom system with clearance under periodic excitation as the research object, and analyzed the relationship and matching between dynamic behavior and parameters of the system.

With the deepening of theoretical research, the application of mechanical systems with clearances and impact vibration systems is also rapidly developing. However, these studies are basically theoretical analysis and numerical simulation of single-degree-of-freedom or two-degree-of-freedom systems, and few studies on multi-degree-of-freedom and high-dimensional complex systems. As the degree of freedom increases, the system will have complex bifurcation and singularity problems. Therefore, it is necessary to study the dynamics of multi-degree-of-freedom systems.

In this paper, a three-degree-of-freedom mechanical vibro-impact model with clearance was established. The model is solved by modal analysis method, and the approximate analytical solution of the model is obtained preliminarily. Then, considering the perturbed periodic motion of the model, the expression of periodic motion is derived from the known momentum transformation equation and boundary conditions. According to the expression, the Poincaré mapping and the corresponding linearization matrix are obtained. Finally, the data simulation of the system is carried out, and the characteristic curve of the period doubling bifurcation of the system under certain parameters is made by Matlab and Maple, according to the period doubling bifurcation theorem, and the period doubling bifurcation point of the system is found.

II. PHYSICAL MODEL AND MOTION EQUATION OF VIBRATION COLLISION SYSTEM WITH CLEARANCE

We consider a three-degree-of-freedom collision model with clearance, as shown in Fig. 1.

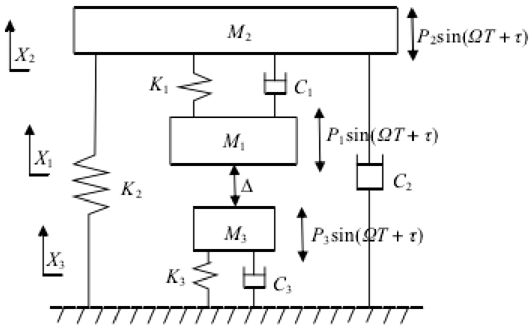

Figure 1. Three-degree-of-freedom vibro-imapct model with clearance

The masses of the three oscillators are M_1, M_2, M_3, respectively, which are connected by the linear spring with stiffness K_1, K_2, K_3 and linear dampers with resistance coefficients C_1, C_2, C_3. Each oscillator moves in the y direction and is subject to periodic forces $P_i\sin(\Omega T+\tau), i=1,2,3$.

The collision occurs when the displacement difference between the oscillator M_1 and the oscillator M_3 is the gap Δ. After collision, the velocities of the three oscillators change, and then they move at new initial velocities, colliding again, forming periodic motion.

Before the oscillator collides, the dimensionless differential equations of M_1, M_2 and M_3 motion can be expressed as follows:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \mu_{m2} & 0 \\ 0 & 0 & \mu_{m3} \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} + 2\xi \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1+\mu_{c2} & 0 \\ 0 & 0 & \mu_{c3} \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{Bmatrix} + \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1+\mu_{k2} & 0 \\ 0 & 0 & \mu_{k3} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} f_{10} \\ f_{20} \\ f_{30} \end{Bmatrix} \sin(\omega t+\tau) \tag{1}$$

When $x_1 - x_3 = \delta$, the impact equation of block M_1, M_2 is as follows:

$$\dot{x}_{1-} + \mu_{m3}\dot{x}_{3-} = \dot{x}_{1+} + \mu_{m3}\dot{x}_{3+}$$

Collision recovery coefficient R satisfies

$$R = \frac{\dot{x}_{3+} - \dot{x}_{1+}}{\dot{x}_{1+} - \dot{x}_{3+}}$$

Where $\dot{x}_{1-}, \dot{x}_{1+}, \dot{x}_{3-}, \dot{x}_{3+}$ denote the impact velocity in the forward and afterward collision time.

Dimensionless constants are as follows:

$$\mu_{mi} = \frac{M_i}{M_1}, \mu_{ci} = \frac{C_i}{C_1}, \mu_{ki} = \frac{K_i}{K_1}, \xi = \frac{C_1}{2\sqrt{K_1 M_1}},$$

$$x_i = \frac{X_i K_1}{P_0}, f_{i0} = \frac{P_i}{P_0}, \omega = \Omega\sqrt{\frac{M_1}{K_1}}, t = T\sqrt{\frac{M_1}{K_1}},$$

$$\delta = \frac{\Delta \cdot K_1}{P_0}, P_0 = \sqrt{P_1^2 + P_2^2 + P_3^2}, i = 1,2,3$$

Where $\dot{x}_i$ and $\ddot{x}_i$ denote the first derivative and the second derivative of the displacement $\ddot{x}_i$ of the oscillator M_i for time t are expressed respectively.

III. MODAL ANALYSIS

Let $q_j(t) = u_j f(t), j = 1,2,3$ be the synchronous solution of the system and u_j be a set of constants.

Its regular modal matrix is:

$$[u] = \left[\{u^{(1)}\}, \{u^{(2)}\}, \{u^{(3)}\}\right] = \begin{bmatrix} u_1^{(1)} & u_1^{(2)} & u_1^{(3)} \\ u_2^{(1)} & u_2^{(2)} & u_2^{(3)} \\ u_3^{(1)} & u_3^{(2)} & u_3^{(3)} \end{bmatrix}$$

Its regular mode equation of motion is as follows:

$$[M_r]\{\ddot{q}(t)\} + [C_r]\{\dot{q}(t)\} + [K_r]\{q(t)\} = \{Q_r(t)\}$$

Where $[M_r]$ represents the regular modal mass, $[C_r]$ represents the regular modal damping, $[K_r]$ represents the regular modal stiffness, and $\{Q_r(t)\}$ represents the generalized excitation force train vector in the regular modal coordinates, that is $\{Q_r(t)\} = [u]^T \{f_{10}(t), f_{20}(t), f_{30}(t)\}^T$.

We assume that the damping in the model is proportional, that is $[c] = \alpha[m] + \beta[k]$. Energy loss occurs during the collision process, which is determined by the collision recovery coefficient R, and the collision duration is ignored.

That is to say, the regular modal damping is:

$$[C_r]=[u]^T[c][u]=\begin{bmatrix} C_1 & 0 & 0 \\ 0 & C_2 & 0 \\ 0 & 0 & C_3 \end{bmatrix}$$
$$=\begin{bmatrix} 2\gamma_1\omega_1 M_1 & 0 & 0 \\ 0 & 2\gamma_2\omega_2 M_2 & 0 \\ 0 & 0 & 2\gamma_3\omega_3 M_3 \end{bmatrix}$$
$$=\begin{bmatrix} 2\gamma_1\omega_1 & 0 & 0 \\ 0 & 2\gamma_2\omega_2 & 0 \\ 0 & 0 & 2\gamma_3\omega_3 \end{bmatrix}$$

Where $\omega_1, \omega_2, \omega_3$ is the natural frequency, $\gamma_1, \gamma_2, \gamma_3$ is the regular modal damping ratio, that i

$$\gamma_r=\frac{\alpha+\beta\omega_r^2}{2\omega_r}, r=1,2,3.$$

IV. Periodic Motion and Poincaré Mapping

By means of modal superposition method [8], the general solution of equation (1) can be obtained:

$$\begin{cases} x_i=\sum_{j=i}^{3} u_i^{(j)}(e^{-\eta_j t}\left(a_j\cos\omega_{dj}t+b_j\sin\omega_{dj}t\right) \\ \quad +A_j\sin\left(\omega t+\tau\right)+B_j\cos\left(\omega t+\tau\right)) \\ \dot{x}_i=\sum_{j=i}^{3} u_i^{(j)}(e^{-\eta_j t}(\left(b_j\omega_{dj}-a_j\eta_j\right)\cos\omega_{dj}t \\ \quad -\left(a_j\omega_{dj}+b_j\eta_j\right)\sin\omega_{dj}t) \\ \quad +A_j\omega\cos\left(\omega t+\tau\right)-B_j\omega\sin\left(\omega t+\tau\right)) \end{cases} \tag{2}$$

Where $\eta_j=\xi\omega_j^2, \omega_{dj}=\sqrt{\omega_j^2-\eta_j^2}, j=1,2,3$, and b_j are the integral constant determined by the initial conditions and modal parameters of the system, A_j and B_j are the amplitude constant, and

$$A_j=\frac{\omega_j^2-\omega^2}{\left(\omega_j^2-\omega^2\right)^2+\left(2\eta_j^2\omega\right)^2}F_j$$

$$B_j=\frac{-2\eta_j\omega}{\left(\omega_j^2-\omega^2\right)^2+\left(2\eta_j^2\omega\right)^2}F_j$$

Where $F_j=\sum_{i=1}^{3}u_i^{(j)}f_{i0}$.

Periodic Conditions of Vibration Motion in System Collision:

$$\begin{aligned} & x_1(0)=x_1(2n\pi/\omega)=x_{10}, x_2(0)=x_2(2n\pi/\omega)=x_{20}, \\ & x_3(0)=x_3(2n\pi/\omega)=x_{30}, \\ & x_1(0)-x_3(0)=\delta, \dot{x}_{2+}(0)=\dot{x}_{2-}(2n\pi/\omega)=\dot{x}_{20}, \\ & \dot{x}_{1+}(0)=\frac{1-\mu_{m3}R}{1+\mu_{m3}}\dot{x}_{1-}(2n\pi/\omega)+\frac{\mu_{m3}(1+R)}{1+\mu_{m3}}\dot{x}_{3-}(2n\pi/\omega), \\ & \dot{x}_{3+}(0)=\frac{1-R}{1+\mu_{m3}}\dot{x}_{1-}(2n\pi/\omega)+\frac{\mu_{m3}-R}{1+\mu_{m3}}\dot{x}_{3-}(2n\pi/\omega). \end{aligned} \tag{3}$$

Based on (2) (3), the equation of impact vibration system can be expressed as follows:

$$\begin{cases} x_i=\sum_{j=i}^{3} u_i^{(j)}(e^{-\eta_j t}\left(a_j\cos\omega_{dj}t+b_j\sin\omega_{dj}t\right) \\ \quad +A_j\sin\left(\omega t+\tau_0\right)+B_j\cos\left(\omega t+\tau_0\right)) \\ \dot{x}_i=\sum_{j=i}^{3} u_i^{(j)}(e^{-\eta_j t}(\left(b_j\omega_{dj}-a_j\eta_j\right)\cos\omega_{dj}t \\ \quad -\left(a_j\omega_{dj}+b_j\eta_j\right)\sin\omega_{dj}t) \\ \quad +A_j\omega\cos\left(\omega t+\tau_0\right)-B_j\omega\sin\left(\omega t+\tau_0\right)) \end{cases}$$

Define section $\sigma=\{(x_1,\dot{x}_1,x_2,\dot{x}_2,x_3,\dot{x}_3,\theta)\}\in R^6\times S$,

$x_1-x_2=\delta, \dot{x}_1=\dot{x}_{1+}, \dot{x}_3=\dot{x}_{3+}, \theta=\omega t, S=R\bmod 2\pi$.

Taking σ as Poincaré cross section, the mapping is established as follows:

$$X'=\tilde{f}(\zeta,X) \tag{4}$$

Where ζ is bifurcation parameter, $X^*=(x_{10},\dot{x}_{1+},x_{20},\dot{x}_{20},\dot{x}_{3+},\tau_0)^T$ represents a fixed point on Poincaré cross section, and satisfy. X and X' satisfy $X=X^*+\Delta X, X'=X^*+\Delta X'$ denote perturbation in motion. From the previous analysis we can derived, when $\tilde{x}_1-\tilde{x}_3\le\delta$, the motion with perturbation of the system can be expressed as follows:

$$\begin{cases} x_i = \sum_{j=i}^{3} u_i^{(j)}(e^{-\eta_j t}\left(a_j \cos\omega_{dj} t + b_j \sin\omega_{dj} t\right) \\ \quad + A_j \sin\left(\omega t + \tau_0 + \Delta\tau\right) + B_j \cos\left(\omega t + \tau_0 + \Delta\tau\right)) \\ \dot{x}_i = \sum_{j=i}^{3} u_i^{(j)}(e^{-\eta_j t}(\left(b_j\omega_{dj} - a_j\eta_j\right)\cos\omega_{dj} t \\ \quad -\left(a_j\omega_{dj} + b_j\eta_j\right)\sin\omega_{dj} t) \\ \quad + A_j\omega\cos\left(\omega t + \tau_0 + \Delta\tau\right) - B_j\omega\sin\left(\omega t + \tau_0 + \Delta\tau\right)) \end{cases}$$

For perturbed motions, we assume that when M_1 and M_2 collide, $t = 0$, when they re-collide, $t_e = \dfrac{2n\pi + \Delta\theta}{\omega}$, $\Delta\theta = \Delta\tau' - \Delta\tau$ .boundary constraints of equation (1) (2) collision motion:

$$\begin{aligned} &\tilde{x}_1(0) = x_{10} + \Delta x_{10}, \dot{\tilde{x}}_1(0) = \dot{x}_{1+} + \Delta\dot{x}_{1+}, \\ &\tilde{x}_1(t_e) = x_{10} + \Delta x'_{10}, \dot{\tilde{x}}_1(t_e) = \dot{x}_{1-} + \Delta\dot{x}'_{1-}, \\ &\tilde{x}_2(0) = x_{20} + \Delta x_{20}, \dot{\tilde{x}}_1(0) = \dot{x}_{20} + \Delta\dot{x}_{20}, \\ &\tilde{x}_2(t_e) = x_{20} + \Delta x'_{20}, \dot{\tilde{x}}_2(t_e) = \dot{x}_{20} + \Delta\dot{x}'_{20}, \\ &\tilde{x}_3(0) = x_{30} + \Delta x_{30}, \dot{\tilde{x}}_3(0) = \dot{x}_{3+} + \Delta\dot{x}_{3+}, \\ &\tilde{x}_3(t_e) = x_{30} + \Delta x'_{30}, \dot{\tilde{x}}_2(t_e) = \dot{x}_{3-} + \Delta\dot{x}'_{3-}, \\ &\tilde{x}_1(0) - \tilde{x}_3(0) = \tilde{x}_1(t_e) - \tilde{x}_3(t_e) = \delta, \\ &\dot{\tilde{x}}_1(t_{e+}) - \mu_{m3}\dot{\tilde{x}}_3(t_{e+}) = \dot{\tilde{x}}_1(t_{e-}) - \mu_{m3}\dot{\tilde{x}}_3(t_{e-}). \end{aligned}$$

By substituting the condition of $t = 0$ into formula (4), the undetermined parameters can be obtained:

$$\begin{aligned} \tilde{a}_1 = \frac{1}{D}&((u_2^{(2)}u_3^{(3)} - u_2^{(3)}u_3^{(2)})(x_{10} + \vartriangle x_{10}) - (u_1^{(2)}u_3^{(3)} - u_1^{(3)}u_3^{(2)}) \\ &(x_{20} + \vartriangle x_{20}) - (u_1^{(3)}u_2^{(2)} - u_1^{(2)}u_2^{(3)})(x_{30} + \vartriangle x_{30}) \\ &- DA_1 \sin(\tau_0 + \vartriangle\tau) - DB_1\cos(\tau_0 + \vartriangle\tau)) \end{aligned}$$

$$\begin{aligned} \tilde{a}_2 = \frac{1}{D}&((u_2^{(3)}u_3^{(1)} - u_2^{(1)}u_3^{(3)})(x_{10} + \vartriangle x_{10}) - (u_1^{(3)}u_3^{(1)} - u_1^{(1)}u_3^{(3)}) \\ &(x_{20} + \vartriangle x_{20}) - (u_1^{(1)}u_2^{(3)} - u_1^{(3)}u_2^{(2)})(x_{30} + \vartriangle x_{30}) \\ &- DA_2 \sin(\tau_0 + \vartriangle\tau) - DB_2\cos(\tau_0 + \vartriangle\tau)) \end{aligned}$$

$$\begin{aligned} \tilde{a}_3 = \frac{1}{D}&((u_2^{(1)}u_3^{(2)} - u_2^{(2)}u_3^{(1)})(x_{10} + \vartriangle x_{10}) \\ &- (u_1^{(1)}u_3^{(2)} - u_1^{(2)}u_3^{(1)})(x_{20} + \vartriangle x_{20}) \\ &- (u_1^{(2)}u_2^{(1)} - u_1^{(1)}u_2^{(2)})(x_{30} + \vartriangle x_{30}) \\ &- DA_1 \sin(\tau_0 + \vartriangle\tau) - DB_1\cos(\tau_0 + \vartriangle\tau)) \end{aligned}$$

$$\begin{aligned} \tilde{b}_1 = \frac{1}{D\omega_{d1}}&((u_2^{(2)}u_3^{(3)} - u_2^{(3)}u_3^{(2)})(\eta_1 x_{10} + \eta_1\vartriangle x_{10} + \dot{x}_{1+} + \vartriangle\dot{x}_{1+}) \\ &-(u_1^{(2)}u_3^{(3)} - u_1^{(3)}u_3^{(2)})(\eta_1 x_{20} + \eta_1\vartriangle x_{20} + \dot{x}_{20} + \vartriangle\dot{x}_{20}) \\ &-(u_1^{(3)}u_2^{(2)} - u_1^{(2)}u_2^{(3)})(\eta_1 x_{30} + \eta_1\vartriangle x_{30} + \dot{x}_{3+} + \vartriangle\dot{x}_{3+}) \\ &-D(A_1\omega + \eta_1 B_1)\cos(\tau_0 + \vartriangle\tau) + D(B_1\omega - \eta_1 A_1)\cos(\tau_0 + \vartriangle\tau)) \end{aligned}$$

$$\begin{aligned} \tilde{b}_2 = \frac{1}{D\omega_{d2}}&((u_2^{(3)}u_3^{(1)} - u_2^{(1)}u_3^{(3)})(\eta_2 x_{10} + \eta_2\vartriangle x_{10} + \dot{x}_{1+} + \vartriangle\dot{x}_{1+}) \\ &-(u_1^{(3)}u_3^{(1)} - u_1^{(1)}u_3^{(3)})(\eta_2 x_{20} + \eta_2\vartriangle x_{20} + \dot{x}_{20} + \vartriangle\dot{x}_{20}) \\ &-(u_1^{(1)}u_2^{(3)} - u_1^{(3)}u_2^{(1)})(\eta_2 x_{30} + \eta_2\vartriangle x_{30} + \dot{x}_{3+} + \vartriangle\dot{x}_{3+}) \\ &-D(A_2\omega + \eta_2 B_2)\cos(\tau_0 + \vartriangle\tau) \\ &+D(B_2\omega - \eta_2 A_2)\cos(\tau_0 + \vartriangle\tau)) \end{aligned}$$

$$\begin{aligned} \tilde{b}_3 = \frac{1}{D\omega_{d1}}&((u_2^{(1)}u_3^{(2)} - u_2^{(2)}u_3^{(1)})(\eta_3 x_{10} + \eta_3\vartriangle x_{10} + \dot{x}_{1+} + \vartriangle\dot{x}_{1+}) \\ &-(u_1^{(1)}u_3^{(2)} - u_1^{(2)}u_3^{(1)})(\eta_3 x_{20} + \eta_3\vartriangle x_{20} + \dot{x}_{20} + \vartriangle\dot{x}_{20}) \\ &-(u_1^{(2)}u_2^{(1)} - u_1^{(1)}u_2^{(2)})(\eta_3 x_{30} + \eta_3\vartriangle x_{30} + \dot{x}_{3+} + \vartriangle\dot{x}_{3+}) \\ &-D(A_3\omega + \eta_3 B_3)\cos(\tau_0 + \vartriangle\tau) \\ &+D(B_3\omega - \eta_3 A_3)\cos(\tau_0 + \vartriangle\tau)) \end{aligned}$$

D is the module of regular modal matrix $[u]$.

By substituting the condition of $t = t_e$ into formula (4), the undetermined parameters can be obtained:

$$\sum_{j=1}^{3} u_1^{(j)}\tilde{v}_j(t_e) - \sum_{j=1}^{3} u_3^{(j)}\tilde{v}_j(t_e) = \delta$$

Its Poincaré mapping $X' = \tilde{f}(\zeta, X)$ is:

$$x_{10} + \vartriangle x'_{10} = \sum_{j=1}^{3} u_1^{(j)}\tilde{v}_j(t_e) \qquad \dot{x}_{20} + \vartriangle\dot{x}'_{20} = \sum_{j=1}^{3} u_2^{(j)}\dot{\tilde{v}}_j(t_e)$$

$$x_{20} + \vartriangle x'_{20} = \sum_{j=1}^{3} u_2^{(j)}\tilde{v}_j(t_e)$$

$$\begin{aligned} \dot{x}_{1+} + \vartriangle\dot{x}'_{1+} &= \frac{1 - \mu_{m3}R}{1 + \mu_{m3}}\sum_{j=1}^{3} u_1^{(j)}\dot{\tilde{v}}_j(t_e) + \\ &\frac{\mu_{m3}(1+R)}{1 + \mu_{m3}}\sum_{j=1}^{3} u_3^{(j)}\dot{\tilde{v}}_j(t_e) \end{aligned}$$

$$\begin{aligned} \dot{x}_{3+} + \vartriangle\dot{x}'_{3+} &= \frac{1 - R}{1 + \mu_{m3}}\sum_{j=1}^{3} u_1^{(j)}\dot{\tilde{v}}_j(t_e) + \\ &\frac{\mu_{m3} - R}{1 + \mu_{m3}}\sum_{j=1}^{3} u_3^{(j)}\dot{\tilde{v}}_j(t_e) \end{aligned}$$

$$\triangle\tau' = \triangle\tau + \triangle\theta(\triangle x_{10}, \triangle\dot{x}_{1+}, \triangle x_{20}, \triangle\dot{x}_{20}, \triangle x_{3+}, \triangle\tau)$$

The expressions of the parameters in the formula are as follows:

$$\begin{aligned}\tilde{v}_j(t) &= e^{-\eta_j t}\left(\tilde{a}_j\cos\omega_{dj}t + \tilde{b}_j\sin\omega_{dj}t\right)\\ &+ A_j\sin\left(\omega t + \tau_0 + \triangle\tau\right)\\ &+ B_j\cos\left(\omega t + \tau_0 + \triangle\tau\right)\end{aligned}$$

$$\begin{aligned}\dot{\tilde{v}}_j(t) &= e^{-\eta_j t}(\left(\tilde{b}_j\omega_{dj} - \tilde{a}_j\eta_j\right)\cos\omega_{dj}t\\ &-\left(\tilde{a}_j\omega_{dj} + \tilde{b}_j\eta_j\right)\sin\omega_{dj}t)\\ &+ A_j\omega\cos\left(\omega t + \tau_0 + \triangle\tau\right)\\ &- B_j\omega\sin\left(\omega t + \tau_0 + \triangle\tau\right)\end{aligned}$$

Define function

$$\begin{aligned}&g(\triangle x_{10}, \triangle\dot{x}_{1+}, \triangle x_{20}, \triangle\dot{x}_{20}, \triangle x_{3+}, \triangle\tau, \triangle\theta)\\ &= \sum_{j=1}^{3} u_1^{(j)}\tilde{v}_j(t_e) - \sum_{j=1}^{3} u_3^{(j)}\tilde{v}_j(t_e) - \delta\end{aligned}$$

The conditions for the existence of periodic motions from fixed points are as follows:

$$g\left(\triangle x_{10}, \triangle\dot{x}_{1+}, \triangle x_{20}, \triangle\dot{x}_{20}, \triangle x_{3+}, \triangle\tau, \triangle\theta\right)\Big|_{(0,0,0,0,0,0,)} = 0$$

Assuming that, according to the existence theorem of implicit functions, we can obtain and satisfy.

Thus, we can get the expression of Poincaré mapping:

$$\begin{aligned}\Delta x_{10}' &= \tilde{f}_1(\Delta x_{10}, \Delta\dot{x}_{1+}, \Delta x_{20}, \Delta\dot{x}_{20}, \Delta x_{3+}, \Delta\tau, \Delta\theta) - x_{10}\\ &= h_1(\Delta x_{10}, \Delta\dot{x}_{1+}, \Delta x_{20}, \Delta\dot{x}_{20}, \Delta x_{3+}, \Delta\tau)\\ \Delta x_{1+}' &= \tilde{f}_2(\Delta x_{10}, \Delta\dot{x}_{1+}, \Delta x_{20}, \Delta\dot{x}_{20}, \Delta x_{3+}, \Delta\tau, \Delta\theta) - \dot{x}_{1+}\\ &= h_2(\Delta x_{10}, \Delta\dot{x}_{1+}, \Delta x_{20}, \Delta\dot{x}_{20}, \Delta x_{3+}, \Delta\tau)\\ \Delta x_{20}' &= \tilde{f}_3(\Delta x_{10}, \Delta\dot{x}_{1+}, \Delta x_{20}, \Delta\dot{x}_{20}, \Delta x_{3+}, \Delta\tau, \Delta\theta) - x_{20}\\ &= h_3(\Delta x_{10}, \Delta\dot{x}_{1+}, \Delta x_{20}, \Delta\dot{x}_{20}, \Delta x_{3+}, \Delta\tau)\\ \Delta\dot{x}_{20}' &= \tilde{f}_4(\Delta x_{10}, \Delta\dot{x}_{1+}, \Delta x_{20}, \Delta\dot{x}_{20}, \Delta x_{3+}, \Delta\tau, \Delta\theta) - \dot{x}_{20}\\ &= h_4(\Delta x_{10}, \Delta\dot{x}_{1+}, \Delta x_{20}, \Delta\dot{x}_{20}, \Delta x_{3+}, \Delta\tau)\\ \Delta x_{3+}' &= \tilde{f}_5(\Delta x_{10}, \Delta\dot{x}_{1+}, \Delta x_{20}, \Delta\dot{x}_{20}, \Delta x_{3+}, \Delta\tau, \Delta\theta) - x_{3+}\\ &= h_5(\Delta x_{10}, \Delta\dot{x}_{1+}, \Delta x_{20}, \Delta\dot{x}_{20}, \Delta x_{3+}, \Delta\tau)\\ \Delta\tau' &= \Delta\theta(\Delta x_{10}, \Delta\dot{x}_{1+}, \Delta x_{20}, \Delta\dot{x}_{20}, \Delta x_{3+}, \Delta\tau, \Delta\theta) + \Delta\tau\\ &= h_6(\Delta x_{10}, \Delta\dot{x}_{1+}, \Delta x_{20}, \Delta\dot{x}_{20}, \Delta x_{3+}, \Delta\tau)\end{aligned}$$

Simplify this mapping to

$$\Delta X' = \tilde{f}(\zeta, X) - X^* = H(\zeta, \Delta X) \qquad (H(\zeta, 0) = 0)$$

Where

$$\Delta X = (\triangle x_{10}, \triangle\dot{x}_{1+}, \triangle x_{20}, \triangle\dot{x}_{20}, \triangle x_{3+}, \triangle\tau)^T$$

$$\Delta X' = (\triangle x_{10}', \triangle\dot{x}_{1+}', \triangle x_{20}', \triangle\dot{x}_{20}', \triangle x_{3+}', \triangle\tau')^T$$

$$H(\zeta, \Delta X) = (h_1, h_2, h_3, h_4, h_5, h_6)\,.$$

Therefore, the Jacobi matrix mapping at the fixed point can be abbreviated as $D_X\tilde{f}(\zeta, X^*) = DH(\zeta, 0)$.

Represent $\Delta X = (\triangle x_{10}, \triangle\dot{x}_{1+}, \triangle x_{20}, \triangle\dot{x}_{20}, \triangle\dot{x}_{3+}, \triangle\tau)^T$ with $\Delta X = (\triangle x_1, \triangle x_2, \triangle x_3, \triangle x_4, \triangle x_5, \triangle x_6)^T$, we can obtain: $\dfrac{\partial h_i}{\partial\triangle x_i} = \dfrac{\partial\tilde{f}_i}{\partial\triangle x_i} - \dfrac{\partial\tilde{f}_i}{\partial\triangle\theta}\cdot\dfrac{\partial g}{\partial\triangle x_i}\Big/\dfrac{\partial g}{\partial\triangle\theta}$.

Let

$$\begin{aligned}&a_{11} = u_2^{(2)}u_3^{(3)} - u_2^{(3)}u_3^{(2)}, a_{12} = -(u_1^{(2)}u_3^{(3)} - u_1^{(3)}u_3^{(2)}),\\ &a_{13} = -(u_1^{(3)}u_2^{(2)} - u_1^{(2)}u_2^{(3)}), a_{21} = u_2^{(3)}u_3^{(1)} - u_2^{(1)}u_3^{(3)},\\ &a_{22} = -(u_1^{(3)}u_3^{(1)} - u_1^{(1)}u_3^{(3)}), a_{23} = -(u_1^{(1)}u_2^{(3)} - u_1^{(3)}u_2^{(2)}),\\ &a_{31} = u_2^{(1)}u_3^{(2)} - u_2^{(2)}u_3^{(1)}, a_{32} = -(u_1^{(1)}u_3^{(2)} - u_1^{(2)}u_3^{(1)}),\\ &a_{33} = -(u_1^{(2)}u_2^{(1)} - u_1^{(1)}u_2^{(2)}), b_{11} = u_2^{(2)}u_3^{(3)} - u_2^{(3)}u_3^{(2)},\end{aligned}$$

$$\begin{aligned}&b_{12} = -(u_1^{(2)}u_3^{(3)} - u_1^{(3)}u_3^{(2)}), b_{13} = -(u_1^{(3)}u_2^{(2)} - u_1^{(2)}u_2^{(3)}),\\ &b_{21} = u_2^{(3)}u_3^{(1)} - u_2^{(1)}u_3^{(3)}, b_{22} = -(u_1^{(3)}u_3^{(1)} - u_1^{(1)}u_3^{(3)}),\\ &b_{23} = -(u_1^{(1)}u_2^{(3)} - u_1^{(3)}u_2^{(1)}), b_{31} = u_2^{(1)}u_3^{(2)} - u_2^{(2)}u_3^{(1)},\\ &b_{32} = -(u_1^{(1)}u_3^{(2)} - u_1^{(2)}u_3^{(1)}), b_{33} = -(u_1^{(2)}u_2^{(1)} - u_1^{(1)}u_2^{(2)}).\end{aligned}$$

So the Jacobi matrix [9] is

$$\begin{aligned}&D_X\tilde{f}(\zeta, X^*) = DH(\zeta, 0)\\ &= \begin{bmatrix} M_{11} & N_{11} & M_{12} & N_{12} & N_{13} & Q_1\\ E_{11} & \dot{E}_{11} & E_{12} & \dot{E}_{12} & \dot{E}_{13} & I_{13}\\ M_{21} & N_{21} & M_{22} & N_{22} & N_{23} & Q_2\\ U_{21} & \dot{U}_{21} & U_{22} & \dot{U}_{22} & \dot{U}_{23} & P_2\\ S_{31} & \dot{S}_{31} & S_{32} & \dot{S}_{32} & \dot{S}_{33} & I_{33}\\ L_{61} & \dot{L}_{61} & L_{62} & \dot{L}_{62} & \dot{L}_{63} & I_{66} \end{bmatrix}_{(\zeta,0,0,0,0,0,0)}\end{aligned}$$

Where

$$\alpha_j = e^{-\eta_j t_e}\left(-\tilde{a}_j \sin\omega_{dj}t_e + \tilde{b}_j \cos\omega_{dj}t_e + \tilde{a}_j \cos\omega_{dj}t_e + \tilde{b}_j \sin\omega_{dj}t_e\right)$$
$$+ A_j \cos\left(\triangle\theta + \tau_0 + \triangle\tau\right) - B_j \sin\left(\triangle\theta + \tau_0 + \triangle\tau\right)$$

$$\dot{\alpha}_j = e^{-\eta_j t_e}(\left(\tilde{b}_j\omega_{dj} - \tilde{a}_j\eta_j\right)\left(-\sin\omega_{dj}t_e + \cos\omega_{dj}t_e\right)$$
$$-\left(\tilde{a}_j\omega_{dj} + \tilde{b}_j\eta_j\right)\left(\cos\omega_{dj}t_e + \sin\omega_{dj}t_e\right))$$
$$-A_j\omega\sin\left(\triangle\theta + \tau_0 + \triangle\tau\right) + B_j\omega\cos\left(\triangle\theta + \tau_0 + \triangle\tau\right)$$

$$\beta_{jq} = e^{-\eta_j t_e}\left(a_{jq}\cos\omega_{dj}t_e + \frac{1}{\omega_{dj}}b_{jq}\eta_j\sin\omega_{dj}t_e\right)$$

$$\dot{\beta}_{jq} = e^{-\eta_j t_e}\frac{1}{\omega_{dj}}b_{jq}\sin\omega_{dj}t_e$$

$$\psi_{jq} = e^{-\eta_j t_e}(\left(b_{jq}\eta_j - a_{jq}\eta_j\right)\cos\omega_{dj}t_e$$
$$-\left(a_{jq}\omega_{dj} + \frac{1}{\omega_{dj}}b_{jq}\eta_j^2\right)\sin\omega_{dj}t_e)$$

$$\dot{\psi}_{jq} = e^{-\eta_j t_e}\left(b_{jq}\eta_j\cos\omega_{dj}t_e - \frac{1}{\omega_{dj}}b_{jq}\eta_j^2\sin\omega_{dj}t_e\right)$$

$$\varphi_j = e^{-\eta_j t_e}(\left(-A_j\cos(\tau_0 + \triangle\tau) + B_j\sin(\tau_0 + \triangle\tau)\right)\cos\omega_{dj}t_e$$
$$+((A_j\omega + \eta_j B_j)\sin(\tau_0 + \triangle\tau)$$
$$-(B_j\omega - \eta_j A_j)\cos(\tau_0 + \triangle\tau))\sin\omega_{dj}t_e)$$
$$+A_i\cos(\tau_0 + \triangle\tau) - B_i\sin(\tau_0 + \triangle\tau)$$

$$\dot{\varphi}_j = e^{-\eta_j t_e}((\ ((A_j\omega + \eta_j B_j)\sin(\tau_0 + \triangle\tau)$$
$$-(B_j\omega - \eta_j A_j)\sin(\tau_0 + \triangle\tau))\ \omega_{dj}$$
$$-\left(-A_j\cos(\tau_0 + \triangle\tau) + B_j\sin(\tau_0 + \triangle\tau)\right)\eta_j)\cos\omega_{dj}t_e$$
$$-(\left(-A_j\cos(\tau_0 + \triangle\tau) + B_j\sin(\tau_0 + \triangle\tau)\right)\omega_{dj}$$
$$+((A_j\omega + \eta_j B_j)\sin(\tau_0 + \triangle\tau)$$
$$-(B_j\omega - \eta_j A_j)\sin(\tau_0 + \triangle\tau))\ \eta_j)\sin\omega_{dj}t_e)$$
$$-A_i\omega\cos(\tau_0 + \triangle\tau) - B_i\omega\sin(\tau_0 + \triangle\tau)$$

$$W = \sum_{j=1}^{3}\left(u_1^{(j)} - u_3^{(j)}\right)(e^{-\eta_j t_e}(-\tilde{a}_j\sin\omega_{dj}t_e + \tilde{b}_j\cos\omega_{dj}t_e$$
$$+\tilde{a}_j\cos\omega_{dj}t_e + \tilde{b}_j\sin\omega_{dj}t_e)$$
$$+A_j\cos\left(\triangle\theta + \tau_0 + \triangle\tau\right) - B_j\sin\left(\triangle\theta + \tau_0 + \triangle\tau\right))$$

$\frac{1}{D}a_{jq}, \frac{\eta_j}{D\omega_{dj}}b_{jq}$ represent the coefficient of $\triangle x_q$ in the integral constant.

$$P_p = \sum_{j=1}^{3}u_p^{(j)}\left(\dot{\varphi}_j - \dot{\alpha}_j \cdot \frac{\sum_{j=1}^{3}\left(u_1^{(j)} - u_3^{(j)}\right)\varphi_j}{W}\right)$$

$$P_p = \sum_{j=1}^{3}u_p^{(j)}\left(\dot{\varphi}_j - \dot{\alpha}_j \cdot \frac{\sum_{j=1}^{3}\left(u_1^{(j)} - u_3^{(j)}\right)\varphi_j}{W}\right)$$

$$Q_p = \sum_{j=1}^{3}u_p^{(j)}\left(\varphi_j - \alpha_j \cdot \frac{\sum_{j=1}^{3}\left(u_1^{(j)} - u_3^{(j)}\right)\varphi_j}{W}\right)$$

$$M_{pq} = \frac{1}{D}\sum_{j=1}^{3}u_p^{(j)}\left(\beta_{jq} - \alpha_j \cdot \frac{\sum_{j=1}^{3}\left(u_1^{(j)} - u_3^{(j)}\right)\beta_{jq}}{W}\right)$$

$$N_{pq} = \frac{1}{D}\sum_{j=1}^{3}u_p^{(j)}\left(\dot{\beta}_{jq} - \dot{\alpha}_j \cdot \frac{\sum_{j=1}^{3}\left(u_1^{(j)} - u_3^{(j)}\right)\beta_{jq}}{W}\right)$$

$$U_{pq} = \frac{1}{D}\sum_{j=1}^{3}u_p^{(j)}\left(\psi_{jq} - \alpha_j \cdot \frac{\sum_{j=1}^{3}\left(u_1^{(j)} - u_3^{(j)}\right)\beta_{jq}}{W}\right)$$

$$\dot{U}_{pq} = \frac{1}{D}\sum_{j=1}^{3}u_p^{(j)}\left(\dot{\psi}_{jq} - \dot{\alpha}_j \cdot \frac{\sum_{j=1}^{3}\left(u_1^{(j)} - u_3^{(j)}\right)\beta_{jq}}{W}\right)$$

$$E_{1q}=\frac{1-\mu_{m3}R}{1+\mu_{m3}}U_{1q}+\frac{\mu_{m3}(1+R)}{1+\mu_{m3}}U_{3q}$$

$$\dot{E}_{1q}=\frac{1-\mu_{m3}R}{1+\mu_{m3}}\dot{U}_{1q}+\frac{\mu_{m3}(1+R)}{1+\mu_{m3}}\dot{U}_{3q}$$

$$I_{13}=\frac{1-\mu_{m3}R}{1+\mu_{m3}}P_1+\frac{\mu_{m3}(1+R)}{1+\mu_{m3}}P_3$$

$$S_{3q}=\frac{1-R}{1+\mu_{m3}}U_{1q}+\frac{\mu_{m3}-R}{1+\mu_{m3}}U_{3q}$$

$$\dot{S}_{3q}=\frac{1-R}{1+\mu_{m3}}\dot{U}_{1q}+\frac{\mu_{m3}-R}{1+\mu_{m3}}\dot{U}_{3q}$$

$$I_{33}=\frac{1-R}{1+\mu_{m3}}P_1+\frac{\mu_{m3}-R}{1+\mu_{m3}}P_3$$

$$I_{66}=1-\omega^2\cdot\frac{\sum_{j=1}^{3}\left(u_1^{(j)}-u_3^{(j)}\right)\varphi_j}{W}$$

$$L_{6q}=\frac{\sum_{j=1}^{3}\left(u_1^{(j)}-u_3^{(j)}\right)\beta_{jq}}{W}$$

$$\dot{L}_{6q}=\frac{\sum_{j=1}^{3}\left(u_1^{(j)}-u_3^{(j)}\right)\dot{\beta}_{jq}}{W}$$

V. Existence of Period-doubling Bifurcations in Impact Vibration Systems

Assume that a fixed point of map is

$$X^*=(x_{10},\dot{x}_{10},\dot{x}_{2+},x_{30},\dot{x}_{30},\tau_0)^T$$

and the characteristic polynomial of the linearized matrix $D_x\tilde{f}(\zeta,X^*)$ at the fixed point is:

$$f_\zeta(\lambda)=\lambda^6+a_1\lambda^5+a_2\lambda^4+a_3\lambda^3+a_4\lambda^2+a_5\lambda+a_6$$

Where $a_i=a_i(\zeta)$.

Lemma 1[10]: If the characteristic polynomial coefficient of $D_X\tilde{f}(\zeta,X^*)$ satisfies the following conditions at the bifurcation point $\zeta=\zeta_0$:

A. Eigenvalue assignment:

$$g_1(\zeta)=1-a_1+a_2-a_3+a_4-a_5+a_6=0 \tag{5}$$

$$g_2(\zeta)=1+a_1+a_2+a_3+a_4+a_5+a_6>0 \tag{6}$$

$$g_3(\zeta)=\begin{vmatrix}1-a_2 & a_1-a_3 & a_2-a_4 & a_3-a_5 & a_4-a_6\\ -a_3 & 1-a_4 & a_1-a_5 & a_2-a_6 & a_3\\ -a_4 & -a_5 & 1-a_6 & a_1 & a_2\\ -a_5 & -a_6 & 0 & 1 & a_1\\ -a_6 & 0 & 0 & 0 & 1\end{vmatrix}>0 \tag{7}$$

$$g_4(\zeta)=\begin{vmatrix}1+a_2 & a_1+a_3 & a_2+a_4 & a_3+a_5 & a_4+a_6\\ a_3 & 1+a_4 & a_1+a_5 & a_2+a_6 & a_3\\ a_4 & a_5 & 1+a_6 & a_1 & a_2\\ a_5 & a_6 & 0 & 1 & a_1\\ a_6 & 0 & 0 & 0 & 1\end{vmatrix}>0 \tag{8}$$

$$\begin{aligned}g_5(\zeta)&=1-a_5-a_3-2a_6^2-a_5^2-a_4^2+a_3a_6+a_3a_5\\&+a_2a_5+a_1a_6+a_1a_4+a_5a_6^2+a_5^3-2a_4a_5a_6+a_3a_6^2\\&-a_3a_5a_6-a_2a_5^2+2a_2a_4a_6+2a_1a_5a_6+a_1a_4a_5\\&-a_1a_3a_6-2a_1a_2a_6-a_1^2a_5+a_6^4-a_3a_6^3+a_2a_5a_6^2\\&-a_2^2a_6^2+a_1a_6^3-a_1a_5^2a_6+a_1a_4a_6^2+a_1a_3a_6^2\\&+a_1a_2a_5a_6-a_1^2a_6^2-a_1^2a_4a_6+a_1^3a_6>0\end{aligned} \tag{9}$$

$$\begin{aligned}g_6(\zeta)&=1+a_5+a_3-2a_6^2-a_5^2-a_4^2-a_3a_6+a_3a_5\\&-a_2a_5-a_1a_6-a_1a_4-a_5a_6^2-a_5^3+2a_4a_5a_6-a_3a_6^2\\&-a_3a_5a_6-a_2a_5^2+2a_2a_4a_6+2a_1a_5a_6+a_1a_4a_5\\&-a_1a_3a_6+2a_1a_2a_6+a_1^2a_5+a_6^4+a_3a_6^3-a_2a_5a_6^2\\&-a_2^2a_6^2-a_1a_6^3+a_1a_5^2a_6-a_1a_4a_6^2+a_1a_3a_6^2\\&+a_1a_2a_5a_6-a_1^2a_6^2-a_1^2a_4a_6-a_1^3a_6>0\end{aligned} \tag{10}$$

$$g_7(\zeta)=1-a_5-a_6^2+a_1a_6>0 \tag{11}$$

$$g_8(\zeta)=1+a_5-a_6^2-a_1a_6>0 \tag{12}$$

B. Transversality condition

$$h(\zeta)=\frac{-a_1'+a_2'-a_3'+a_4'-a_5'+a_6'}{-6+5a_1-4a_2+3a_3-2a_4+a_5}\neq 0 \tag{13}$$

VI. Numerical simulation

Let

$$\mu_{m2}=1,\mu_{m3}=9,\mu_{k2}=1,\mu_{k3}=2,$$
$$\mu_{c3}=2,\delta=0.01,f_{10}=0.2, \quad ,$$
$$f_{20}=0.2,f_{30}=0.2,R=0.8,\omega=2$$

and its regular modal matrix is:

$$\begin{bmatrix} 0 & -0.8507 & -0.5257 \\ 0 & -0.5257 & 0.8507 \\ 0.3333 & 0 & 0 \end{bmatrix}$$

Natural frequencies ω_1 , ω_2 , ω_3 are 0.4714, 0.6180 and 1.6180, respectively.

$$\eta = \left[0.2222\xi, 0.3820\xi, 2.6180\xi\right]$$

$$\omega_{dj} = \begin{bmatrix} \sqrt{0.2222 - 0.0494\xi^2} \\ \sqrt{0.3820 - 0.1459\xi^2} \\ \sqrt{2.6180 - 6.8541\xi^2} \end{bmatrix}^T$$

$$F = [0.0667, -0.2753, 0.0650]^T$$

According to Lemma 1, we study the period doubling bifurcation behavior of a collisional vibration system.

As shown in Fig. 2, when $\zeta = \zeta_0 = 0.33425$, condition (5) ensures that there is a real eigenvalue, condition (6-12) ensures that other eigenvalues are in the unit circle, condition (13), $h(\zeta) = h(\zeta_0) = 0.66581 \neq 0$ ensures that the velocity is not zero when the eigenvalue and its parameters change across the unit circle.

So the period doubling bifurcation will occur when $\zeta = \zeta_0 = 0.33425$.

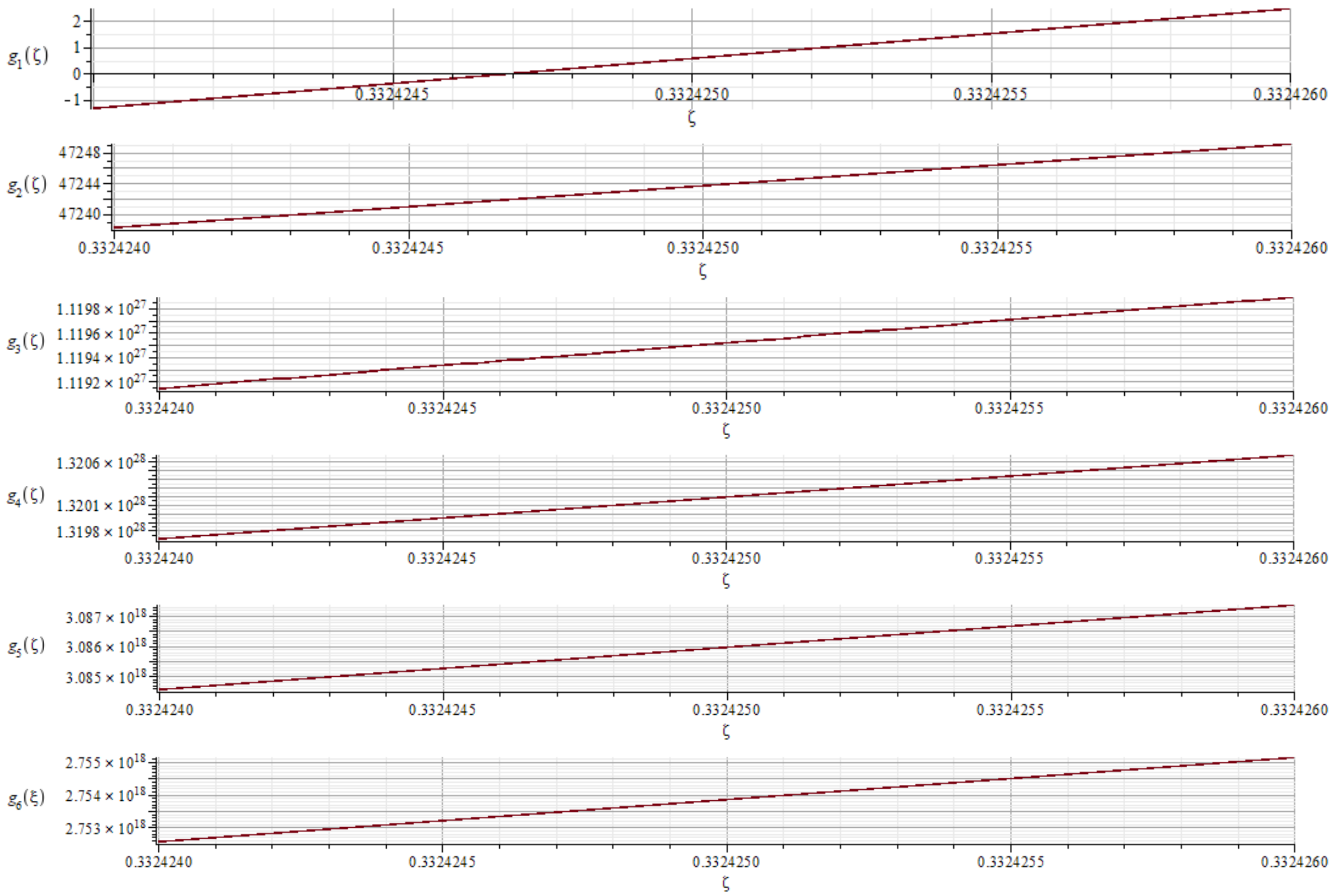

Figure 2. Characteristic curve of double period bifurcation

VII. CONCLUTION

In this paper, the theoretical model of a three-degree-of-freedom mechanical impact system with clearance is studied, and the regular modal matrix of the system is obtained; the general solution of the system equation is obtained by means of modal superposition method; the Poincaré mapping is established, and the characteristic polynomial is obtained by means of Jacobi matrix of Poincaré mapping at the fixed point, and then the parameters of the bifurcation are discriminated by using the double-period bifurcation theorem. Finally, numerical simulation is carried out by Maple to verify the theoretical analysis results.

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