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A Bi-operational Alphabet Approach to Solve a Linear Programming Problem

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Abstract-In this paper we deal with a resolution of a Linear Programming Problem (LPP) using a Bi-operational alphabet.

Keywords- *Bi-Operational Alphabet, Linear Programming, LPP*

I. Introduction

In classical cryptography, the Lester S. Hill cipher is a polygraphic substitution cipher based on linear algebra, especially on an elementary knowledge of matrices operations. The author is Lester S. Hill in 1929 [1]. It was the first [2] polygraphic cipher in which to operate on more than three symbols at once was handy.

Thus, the cryptography, the real science governing information coding, has grown even more with the advent of communications systems modern (Internet, etc...) where there is a need absolute to protect the data exchanged individuals. The context is that of secret messages. It's a subject that is very rich in mathematical applications and is very interested in communications and e-commerce.

Many authors worked on cryptography [3, 4, 5, 6] and did many applications on others sciences.

In LLP, the constraints and the objective function are linear expressions [7] in some unknown variables. In 1947, G. Bernard Dantzig, the Father of the linear programming, formulated the general LPP and gave the general form of the mathematical programming problem:

Optimize Z= g (x_j), j= 1, … , n

subject to $h_i(x_j) \leq 0$, i = 1, … $x_j \geq 0$,

where the functions g (x_j) and $h_i(x_j)$ are continuous or continuously differentiable.

There are a variety of methods to solve the LPP such as graphical method, simplex method, revised simplex method, etc.

The graphical method is used to a LLP which contains only two unknown variables. The simplex method is applied to general LPP which contains the variables great or equal to two. G.B. Dantzig revised the simplex method in order to make it extremely helpful while using a digital computer. Comparatively for hand computation, the ordinary simplex method is better than the revised simplex method. With this last one, the process requires the use of less memory cells.

Let $a_1, a_2, \dots, a_{26}$ denote any permutation of the letters of the French alphabet.

Let associate the letter a_k with the integer k. we define operations of modular subtraction (modula 26) over the alphabet:

a_k - a_l = a_m , a_l - a_k = a_n,

where m and n are the remainders obtained upon dividing the integers k – l and l - k by the integer 26.

We have the following proposition:

a_m = 26 - a_n, i.e. $(a_m + a_n)$ (modulo 26) = 0.

Lester S. Hill [1] defined the operations of modular addition and multiplication (modulo 26) over the alphabet:

$a_k + a_l = a_m$, $a_k\, a_l = a_n$, where m and n are respectively the remainders obtained upon dividing the integers k + l and kl by the integer 26. The integers k and l are the same or different.

In this paper we solve a LLP using especially a Bi-operational alphabet.

II. Illustration about Bi- Operational Alphabet

Let the letters of the French alphabet be associated with integers as follows:

TABLE I. FRENCH ALPHABET ASSOCIATED WITH INTEGERS

a_1	a	0
a_2	b	1
a_3	c	2
a_4	d	3
a_5	e	4
a_6	f	5
a_7	g	6
a_8	h	7
a_9	i	8
a_{10}	j	9
a_{11}	k	10
a_{12}	l	11
a_{13}	m	12
a_{14}	n	13
a_{15}	o	14
a_{16}	p	15
a_{17}	q	16
a_{18}	r	17
a_{19}	s	18
a_{20}	t	19
a_{21}	u	20
a_{22}	v	21
a_{23}	w	22
a_{24}	x	23
a_{25}	y	24
a_{26}	z	25

We have that

TABLE II. ILLUSTRATION

a - b= z	a + b= b	a b= a
b - a = b	b + a = b	b a= a
c - x= f	c + x = z	c x= u
x - c= v	x + c= z	x c= u
-y - w= g	-y + w= y	-y -w= i
w + y= u	w - y= y	w y= i

III. PROBLEM, MATHEMATICAL FORMULATION AND METHODOLOGY

A. Problem

A factory manufactures c products P_1 and P_2 using two raw materials α et β .

Product P_1 costs e BIF and product P_2 costs f BIF. We have a stock of i kg materials α and h kg of materials β .

To produce a unit of product P_1 we need c kg of α and b kg materials material β ; while a unit of product P_2 is obtained from b kg of materials α and c kg of materials β.

Give and solve a production program for the factory that maximizes its profit .

B. Mathematical formulation

Let Z be the objective function and let x_1 and x_2 be respectively the amounts of P_1 and P_2 to produce.

The objective of this problem is to maximize the total profit

Z= e x_1+ f x_2 .

The constraints are

$c\,x_1 + b\,x_2 \;\le\;$ i

b x_1+ c $x_2 \le$ h , $x_1 \ge 0$, $x_2 \ge 0$.

Hence the complete mathematical model is to find real variables x_1 and x_2 with the following LLP:

$$\begin{cases} Max\ \mathrm{Z} = \mathrm{e}\,x_1 + \mathrm{f}\,x_2 \\ \quad subject\ to \\ c\,x_1 + b\,x_2 \le \mathrm{i} \\ bx_1 + c\,x_2 \le \mathrm{h} \\ \quad x_1 \ge 0, x_2 \ge 0. \end{cases} \qquad (1)$$

C. Methodology

To solve this LLP, we have many methods (graphical method, simplex method, etc). For each method we use the Bi-operational alphabet. In this paper, we choose to use the graphic method because the LLP contains two variables.

IV. RESULTS

The feasible region is a closed bounded convex set having four extreme points.

The maximum of the objective function of the LLP is attained at one of the extreme points of the feasible region.

In order to find the intersection point of the two constraints, we use the Bi-operational alphabet.

$$\begin{cases} c\,x_1 + b\,x_2 = \mathrm{i} \\ b\,x_1 + c\,x_2 = \mathrm{h}. \end{cases} \qquad (2)$$

The determinant D of the system (2) is a primary determinant. Its value is a primary letter:

D = d. , Dx1 = j , Dx2 = g.

Thus, the intersection point is (d, c).

Since the maximum of Z is w which occurs at the point (d, c), the solution to the LLP is

x_1= d,

x_2= c and Max Z = w.

So, by the Bi-operational alphabet, the amounts of P_1 is 3 and the amounts of P_2 is 2 and the maximum profit is 22.

V. ILLUSTRATION USING TORA SOFTWARE

We illustrate the solution with TORA software.

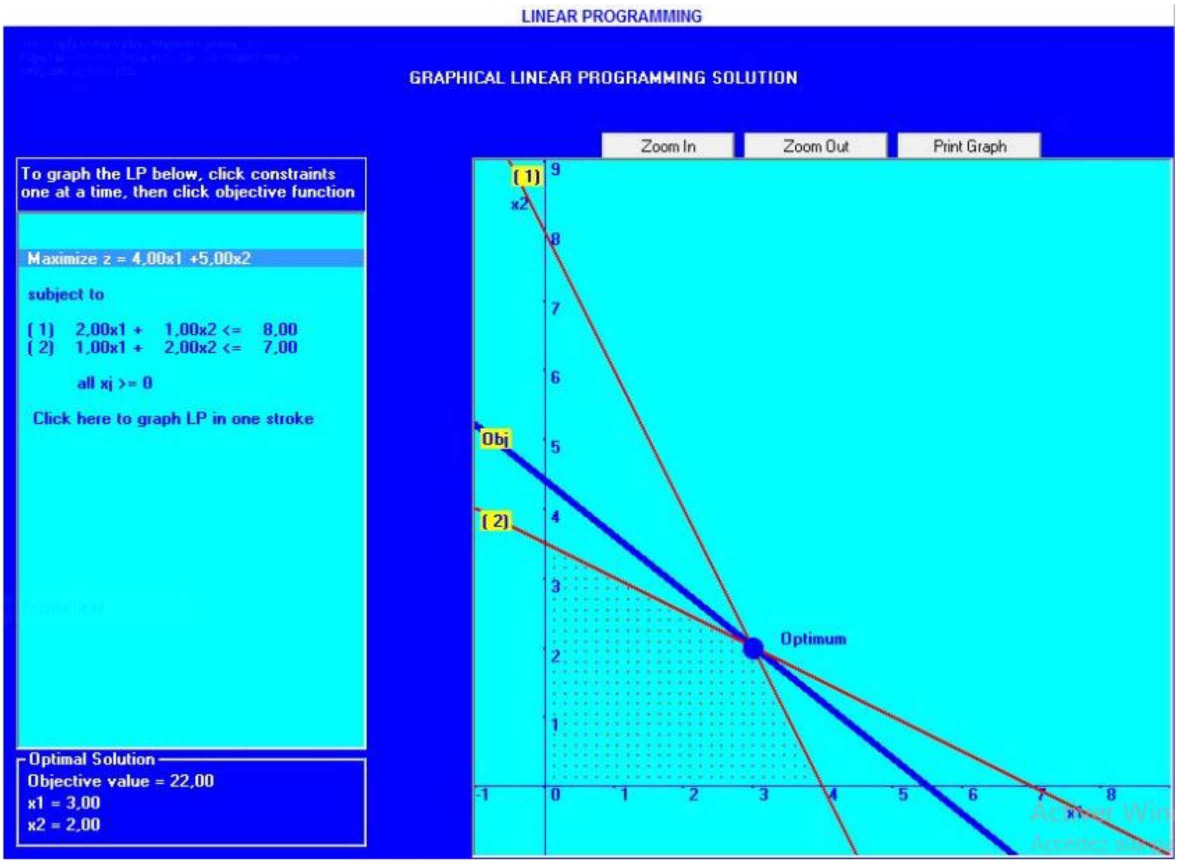

Figure 1. Graphical method with TORA software

Iteration 1					
Basic	x1	x2	sx3	sx4	Solution
z (max)	-4,00	-5,00	0,00	0,00	0,00
sx3	2,00	1,00	1,00	0,00	8,00
sx4	1,00	2,00	0,00	1,00	7,00
Lower Bound	0,00	0,00			
Upper Bound	infinity	infinity			
Unrestr'd (y/n)?	n	n			
Iteration 2					
Basic	x1	x2	sx3	sx4	Solution
z (max)	-1,50	0,00	0,00	2,50	17,50
sx3	1,50	0,00	1,00	-0,50	4,50
x2	0,50	1,00	0,00	0,50	3,50
Lower Bound	0,00	0,00			
Upper Bound	infinity	infinity			
Unrestr'd (y/n)?	n	n			
Iteration 3					
Basic	x1	x2	sx3	sx4	Solution
z (max)	0,00	0,00	1,00	2,00	22,00
x1	1,00	0,00	0,67	-0,33	3,00
x2	0,00	1,00	-0,33	0,67	2,00
Lower Bound	0,00	0,00			
Upper Bound	infinity	infinity			
Unrestr'd (y/n)?	n	n			

Figure 2. Simplex method with TORA software

VI. CONCLUSION AND FUTURE WORK

In this paper, a LLP solution is found using the Bi-operational alphabet. Our results are verified by TORA software which is a suitable tool. In the future work we could solve the LLP by the Revised Simplex Method using the Bi-operational alphabet.

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