Robust Adaptive Observer Design for Lipschitz Class of Nonlinear Systems

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Abstract- This paper addresses parameter and state estimation problem in the presence of the perturbation of observer gain bounded input disturbances for the Lipschitz systems that are linear in unknown parameters and nonlinear in states. A new nonlinear adaptive resilient observer is designed, and its stability conditions based on Lyapunov technique are derived. The gain for this observer is derived systematically using linear matrix inequality approach. A numerical example is provided in which the nonlinear terms depend on unmeasured states. The simulation results are presented to show the effectiveness of the proposed method.

Keywords- *Adaptive observer; linear matrix inequality, nonlinear systems; nonlinear observer; resilient observer; robust estimation.*

I. INTRODUCTION

One of the major difficulties in the design of practical observers for most physical systems are their model uncertainties due to either constant or slow changes of unknown quantities such as unknown physical parameters. Adaptive observers have been used to cope with the lack of knowledge on the system parameters in state estimation problems.

For nonlinear systems with unknown parameters, various adaptive observers have been introduced [1-5]. In [1], the authors reported early results on adaptive observers for nonlinear systems, namely observers estimating the entire state vector using an on-line adaptation for the unknown parameters. The authors in [2-4] focused on a class of nonlinear systems which are transformable by a global parameter-independent state-space diffeomorphism into a system whose dynamics are linear in unmeasured states and nonlinear in inputs and measurable outputs. Then, they designed an adaptive observer for the new system such that the state and parameter estimates both converge asymptotically under the persistence excitation condition. In these works, nonlinear terms are assumed to be related only to the input and the measured output, and disturbances are neglected.

This design method has been extended in [5] and [6] to cover slightly more general case of systems where the nonlinear terms depend on the input and the entire state vector (not just measured outputs) with the nonlinearities satisfying Lipschitz conditions. In this work, a systematic algorithm is provided to check the feasibility of an asymptotically stable adaptive observer. An arbitrarily small disturbance may force the parameter estimates to drift towards infinity, while the state estimation error remains small [7,8]. Several techniques have been introduced to modify the adaptive observer structure to prevent parameter estimation drift. For instance, in [7] and [8], this goal has been achieved by designing robust adaptive observers assuming that the nonlinear terms only depend on the input and the measured outputs.

In [9], a robust-adaptive-observer for sensorless induction-motor drives was designed based on the linearized dynamic equation and linear matrix inequality (LMI) method. The motor's dynamic equations are formulated in the form of a very special class of nonlinear system which are linear in feedforward and nonlinear in the feedback. The stability conditions and the observer gain are obtained by solving the corresponding LMIs. Another LMI-based observer design for a class of Lipschitz nonlinear dynamical systems can be found in [10]. Differential mean value theorem allows the nonlinear error dynamics to be transformed into a linear parameter varying system. The authors introduced a general Lipschitz-like condition on the Jacobian matrix for differentiable systems. To ensure asymptotic convergence of the states estimation error, sufficient conditions are expressed in terms of LMIs. However, for large values of the Lipschitz constant, the stability conditions may become infeasible.

An observer for which the estimation error diverges by a small perturbation in the observer gain is referred to as fragile or non-resilient [11]. Since the observer gains are usually obtained from offline calculations, in many practical applications the gain may have slow drifts; thus, it is necessary that the observer tolerates some perturbations in its coefficients. Authors in [12] have shown that even vanishingly small perturbations in the control coefficients may destabilize the closed–loop system. Afterwards, more researchers concentrated their attention on this subject

[13-17], In [13] an overview of the resilient design technique is presented. In [14] synthesis of a resilient regulator for the linear systems is provided. In [15], a robust resilient Kalman filter design for a class of linear systems with norm-bounded multiplicative uncertainties in the filter gain is introduced. In [16], present an LMI solution for nonlinear resilient observer design is presented. In reference [17], an observer is designed using LMI approach to maintain disturbance attenuation performance in the case of randomly varying perturbations in the observer gain.

In this paper, Lipschitz class of nonlinear systems containing uncertain piecewise constant parameters in the

$$2x^T y \le \varepsilon x^T x + \varepsilon^{-1} y^T y \tag{7}$$

presence of bounded perturbation on the observer gains and bounded exogenous disturbances is considered. Our objectives are to find an LMI-based robust non-fragile adaptive observer for this systems. The proposed observer stabilizes the state estimation error. Moreover, when the persistent excitation condition holds, the parameter estimation vector converges to its true value. Unlike [7], we allow the nonlinear terms in the system depend on the input and all the states, in general, and we modified the adaptive law to overcome some drawback in parameter estimation. Unlike [6], we consider an exogenous input disturbance in the system; also shows that the proposed design is feasible for much larger values of the Lipschitz constants compared to those of the design in [6].

The rest of the paper is organized as follows: Section 2 provides the problem statement. In section 3, the proposed resilient adaptive observer is presented. A numerical example is provided in section 4. Finally, the conclusion remarks are given in section 5.

II. PROBLEM STATEMENT

Consider an uncertain nonlinear system of the form:
$$\dot{x} = Ax + \phi(x,u) + bf(x,u)\theta + \omega$$
$$y = Cx \tag{1}$$

where $x \in \Re^n$, $u \in \Re^q$, $y \in \Re^m$, and $\theta \in \Re^p$ are the state, input, output, and parameter vectors, respectively, $b \in \Re^{n \times m}$, $C \in \Re^{m \times n}$ are constant matrices, $\omega \in \Re^n$ is disturbance input, and $f:[\Re^n \ \Re^q] \to \Re^{m \times p}$, $\phi:\left[\Re^n \ \Re^q\right] \to \Re^n$ are nonlinear functions which are Lipschitz in x with Lipschitz constants γ_1 and γ_2, respectively, i.e.:

$$\|\phi(x_1,u) - \phi(x_2,u)\| < \gamma_1\|x_1 - x_2\| \tag{2}$$
and
$$\|f(x_1,u) - f(x_2,u)\| < \gamma_2\|x_1 - x_2\| \tag{3}$$

for all $x_1, x_2 \in R^n$. System (1) is linear in θ and nonlinear in x with Lipschitz nonlinearities. This is fairly a general class, since, in most cases, nonlinearities are bounded in a Lipschitz manner if the states are bounded [6]. We assume that the

unknown piecewise constant parameter vector and its distance from nominal parameter vector θ_0 are both bounded in the following sense:

$$\|\theta\| \le \gamma_3 \tag{4}$$
$$\|\theta - \theta_0\| \le M \tag{5}$$

and the bounded disturbance ω satisfies following constraint:
$$\|\omega(t)\|_2 \le \rho \tag{6}$$

Lemma 1. [18]: Let x, y be real vectors of the same dimension. Then, for any scalar $\varepsilon > 0$, the following inequality holds:

III. RESILIENT ADAPTIVE OBSERVER DESIGN

If Consider a nonlinear adaptive observer of the form [6]:
$$\dot{\hat{x}} = A\hat{x} + \phi(\hat{x},u) + bf(\hat{x},u)\hat{\theta} + [L + \Delta(t)](y - C\hat{x}) \tag{8}$$

where $\hat{x}$ and $\hat{\theta}$ are the state and parameter estimates, respectively, L is the observer gain and the resilient term $\Delta(t)$ is an additive perturbation on the gain with known bound $\|\Delta(t)\| \le r$ for all t.

Then, the observer error dynamic equation is obtained as:
$$\dot{\tilde{x}} = (A - LC - \Delta C)\tilde{x} + \phi(x,u) - \phi(\hat{x},u) + bf(x,u)\theta$$
$$- bf(\hat{x},u)\hat{\theta} + \omega \tag{9}$$

where $\tilde{x} = x - \hat{x}$ is the state estimation error .

The following theorem provides sufficient conditions for the stability of the robust adaptive observer (8).

Theorem 1: Consider the following parameter adaption law:

$$\dot{\hat{\theta}} = \Gamma^{-1}(f(\hat{x},u)^T C\tilde{x}) - \sigma\Gamma^{-1}(\hat{\theta} - \theta_0) \tag{10}$$

where $\Gamma = \Gamma^T > 0$ is an arbitrary constant matrix and:

$$\sigma = \begin{cases} 0 & \text{if} \quad \|\hat{\theta} - \theta_0\| < M \\ \sigma_0\left(\dfrac{\|\hat{\theta} - \theta_0\|}{M} - 1\right) & \text{if} \quad M \le \|\hat{\theta} - \theta_0\| \le 2M \\ \sigma_0 & \text{if} \quad \|\hat{\theta} - \theta_0\| > 2M \end{cases} \tag{11}$$

with positive constants scalars M and σ_0. If there exist positive real numbers ε_1, ε_2, ε_3 and matrices $P = P^T > 0$ and S, such that $Pb = C^T$ and:

$$\begin{bmatrix} \Lambda & P & P & P & P \\ P & -\varepsilon_1 I & 0 & 0 & 0 \\ P & 0 & -\varepsilon_2 I & 0 & 0 \\ P & 0 & 0 & -\varepsilon_3 I & 0 \\ P & 0 & 0 & 0 & -\varepsilon_4 I \end{bmatrix} < 0 \tag{12}$$

where

$$\Lambda = A^T P - C^T S + PA - S^T C + (\varepsilon_1 \gamma_1^2 + \varepsilon_2 \gamma_2^2 \gamma_3^2 \|b\|^2) I + r^2 \varepsilon_3 C^T C$$

with γ_1, γ_2, and γ_3 defined in (2), (3), and (4), respectively, then the observer gain $L = P^{-1} S^T$ stabilizes the state estimation error dynamics in (9) while the parameter estimation error remains bounded. Moreover, if the following persistency excitation condition holds:

$$\forall t_0, \exists \xi, \delta > 0 \text{ such that :}$$
$$\int_{t_0}^{t_0 + \delta} bf(x(\tau), u(\tau)) f^T(x(\tau), u(\tau)) b^T d\tau > \xi I \qquad (13)$$

then, the parameter estimate vector converges to its true value for all disturbances satisfying $\|\Delta(t)\| \le r$.

Proof: Consider the following Lyapunov function candidate for error dynamic (9):

$$V = \tilde{x}^T P \tilde{x} + \tilde{\theta}^T \Gamma \tilde{\theta} \qquad (14)$$

where $\tilde{\theta} = \theta - \hat{\theta}$ is the parameter estimation error. Taking the derivative of Eq. (14) and using (9), results in:

$$\dot{V} = \tilde{x}^T \left[(A - LC - \Delta C)^T P + P(A - LC - \Delta C) \right] \tilde{x} + 2[\phi(x,u) - \phi(\hat{x},u)]^T P \tilde{x} + 2[bf(x,u)\theta - bf(\hat{x},u)\hat{\theta}]^T P \tilde{x} + 2\tilde{\theta}^T \Gamma \dot{\tilde{\theta}} + 2\tilde{x}^T P \omega \qquad (15)$$

Using Lemma 1 and inequality (2) on the second term, and substituting $\hat{\theta} = \theta - \tilde{\theta}$ in the third term of Eq. (15) result in:

$$\dot{V} \le \tilde{x}^T \left[(A - LC)^T P + P(A - LC) \right] \tilde{x} - 2\tilde{x}^T C^T \Delta^T P \tilde{x} + \tilde{x}^T (\varepsilon_1 \gamma_1^2 + \varepsilon_1^{-1} PP) \tilde{x} + 2[bf(x,u) - bf(\hat{x},u)\theta]^T P \tilde{x} + 2[bf(\hat{x},u)\tilde{\theta}]^T P \tilde{x} + 2\tilde{\theta}^T \Gamma \dot{\tilde{\theta}} + 2\tilde{x}^T P \omega \qquad (16)$$

Again, applying Lemma 1 to the second and the fourth term of inequality (16) with ε_3 and ε_2, respectively, and using (3) and (4) and $\|\Delta(t)\| \le r$ follow that:

$$\dot{V} \le \tilde{x}^T \left[\Omega + \varepsilon_1^{-1} PP + \varepsilon_2^{-1} PP + \varepsilon_3^{-1} PP \right] \tilde{x} + \varepsilon_3 r^2 \tilde{x}^T C^T C \tilde{x} + 2[bf(\hat{x},u)\tilde{\theta}]^T P \tilde{x} + 2\tilde{\theta}^T \Gamma \dot{\tilde{\theta}} + 2\tilde{x}^T P \omega \qquad (17)$$

where

$$\Omega = (A - LC)^T P + P(A - LC) + \varepsilon_1 \gamma_1^2 I + \varepsilon_2 \gamma_2^2 \gamma_3^2 \|b\|^2 I.$$

Since θ is piecewise constant, thus, we assume $\dot{\theta} = 0$ and thus $\dot{\tilde{\theta}} = -\dot{\hat{\theta}}$. Using this fact, substituting (10) in (17), and using $b^T P = C$ yield:

$$\dot{V} \le \tilde{x}^T \left[\Omega + (\varepsilon_1^{-1} + \varepsilon_2^{-1} + \varepsilon_3^{-1}) PP + \varepsilon_3 r^2 C^T C \right] \tilde{x} + 2\sigma \tilde{\theta}^T (\hat{\theta} - \theta_0) + 2\tilde{x}^T P \omega \qquad (18)$$

Then, using (5) follows that:

$$\sigma \tilde{\theta}^T (\hat{\theta} - \theta_0) = \sigma[\theta - \hat{\theta}]^T (\hat{\theta} - \theta_0)$$
$$= \sigma(\theta - \theta_0)^T (\hat{\theta} - \theta_0) - \sigma(\hat{\theta} - \theta_0)^T (\hat{\theta} - \theta_0)$$
$$\le \sigma M \|\hat{\theta} - \theta_0\| - \sigma \|\hat{\theta} - \theta_0\|^2$$
$$\le N \qquad (19)$$

where:

$$N = \sigma \|\hat{\theta} - \theta_0\| \left(M - \|\hat{\theta} - \theta_0\| \right) \qquad (20)$$

If condition (11) holds, the derived upper bound N in (19) is always is non positive, because:

For $\|\hat{\theta} - \theta_0\| < M$, since $\sigma = 0$, then $N = 0$.

For $M \le \|\hat{\theta} - \theta_0\| \le 2M$, we have

$$N = -\frac{\sigma_0}{M} \|\hat{\theta} - \theta_0\| \left(M - \|\hat{\theta} - \theta_0\| \right)^2 \le 0.$$

For $\|\hat{\theta} - \theta_0\| > 2M$, we have $N \le -\sigma_0 M \|\hat{\theta} - \theta_0\| \le 0$.

Therefore, it follows that:

$$2\sigma \tilde{\theta}^T (\hat{\theta} - \theta_0) \le 0 \qquad (21)$$

Substituting the above inequality in (18), it reduced to:

$$\dot{V} \le \tilde{x}^T \left[\Omega + (\varepsilon_1^{-1} + \varepsilon_2^{-1} + \varepsilon_3^{-1}) PP + \varepsilon_3 r^2 C^T C \right] \tilde{x} + 2\tilde{x}^T P \omega \qquad (22)$$

Using Lemma 1 in second part of RHS (22), results in:

$$\dot{V} \le \tilde{x}^T \left[\Omega + (\varepsilon_1^{-1} + \varepsilon_2^{-1} + \varepsilon_3^{-1}) PP + \varepsilon_3 r^2 C^T C \right] \tilde{x} + \varepsilon_4^{-1} \tilde{x}^T PP \tilde{x} + \varepsilon_4 \omega^T \omega \qquad (23)$$

If there exists a positive-definite matrix such that the following inequality holds:

$$\Omega + (\varepsilon_1^{-1} + \varepsilon_2^{-1} + \varepsilon_3^{-1} + \varepsilon_4^{-1}) PP + \varepsilon_3 r^2 C^T C < -\alpha I \qquad (24)$$

then, condition (23) reduces to:

$$\dot{V} \le -\alpha \tilde{x}^T \tilde{x} + \varepsilon_4 \omega^T \omega \qquad (25)$$

Integrating both sides of inequality (25) from $t = 0$ to $t = t_f$ follows that:

$$V(t_f) - V(0) \le -\alpha \int_0^{t_f} \tilde{x}^T(\tau) \tilde{x}(\tau) d\tau + \varepsilon_4 \int_0^{t_f} \omega^T(\tau) \omega(\tau) d\tau \qquad (26)$$

which implies that:

$$\int_0^{t_f} \tilde{x}^T(\tau) \tilde{x}(\tau) d\tau \le \frac{V(0)}{\alpha} + \frac{\varepsilon_4}{\alpha} \int_0^{t_f} \omega^T(\tau) \omega(\tau) d\tau \qquad (27)$$

Thus the robust performance is guaranteed where $\frac{\varepsilon_4}{\alpha}$ is a desired disturbance attenuation level.

Then, using Schur complement Lemma, inequality (24), can be rewrite as:

$$\begin{bmatrix} \Omega+r^2\varepsilon_3 C^T C & P & P & P & P \\ P & -\varepsilon_1 I & 0 & 0 & 0 \\ P & 0 & -\varepsilon_2 I & 0 & 0 \\ P & 0 & 0 & -\varepsilon_3 I & 0 \\ P & 0 & 0 & 0 & -\varepsilon_4 I \end{bmatrix} < 0 \qquad (28)$$

Thus LMI (12) is obtained where $S = L^T P$ and $\Lambda = \Omega + r^2\varepsilon_3 C^T C$.

From (26), we have:

$$\tilde{x}^T P\tilde{x} + \tilde{\theta}^T \Gamma \tilde{\theta} \le V(0) - \alpha \int_0^{t_f} \tilde{x}^T(\tau)\tilde{x}(\tau)d\tau + \qquad (29)$$

$$\varepsilon_3^{-1} \int_0^{t_f} \omega^T(\tau)\omega(\tau)d\tau$$

This implies that:

$$\tilde{\theta}^T \Gamma \tilde{\theta} \le V(0) + \varepsilon_4 \int_0^{t_f} \omega^T(\tau)\omega(\tau)d\tau \qquad (30)$$

$$\le \tilde{x}^T(0)P\tilde{x}(0) + \tilde{\theta}^T(0)\Gamma\tilde{\theta}(0) + \varepsilon_4\rho^2$$

Using Rayleigh-Ritz inequality gives:

$$\left\| \tilde{\theta} \right\| \le \sqrt{\frac{\lambda_{\max}(P)\left\|\tilde{x}(0)\right\|^2 + \lambda_{\max}(\Gamma)\left\|\tilde{\theta}(0)\right\|^2 + \varepsilon_4\rho^2}{\lambda_{\min}(\Gamma)}} \qquad (31)$$

where $\lambda_{\min}(\cdot)$, $\lambda_{\max}(\cdot)$ denote the minimum and maximum singular values of its argument, respectively. This implies $\tilde{\theta} \in L_\infty$. Following the same procedure (29) from to (31), we can prove that $\tilde{x} \in L_\infty$. Since $V(x,\theta) \ge 0$ and $V(0)$ is finite and $\omega \in L_2$, (26) implies that $\tilde{x} \in L_2$. Moreover, since both $\phi(x,u)$ and $f(x,u)$ are Lipschitz, Eq.(9) yields $\dot{\tilde{x}} \in L_\infty$. With $\tilde{x} \in L_\infty$, $\tilde{x} \in L_2$ and $\dot{\tilde{x}} \in L_\infty$, and using Barbalat's lemma [19] follows that $\lim_{t\to\infty} \tilde{x}(t) = 0$, and consequently, it can be also concluded that $\lim_{t\to\infty} \dot{\tilde{x}}(t) = 0$ and $\lim_{t\to\infty} \omega(t) = 0$. Therefore, considering Eq. (9), we have:

$$\lim_{t\to\infty}\left(bf(x,u)\theta - bf(\hat{x},u)\hat{\theta}\right) = 0 \qquad (32)$$

Since $\lim_{t\to\infty} \hat{x} = x$, Eq. (32) reduces to:

$$\lim_{t\to\infty}\left(bf(x,u)(\theta - \hat{\theta})\right) = 0 \qquad (33)$$

Thus if the persistency excitation condition (13) holds, we can say the parameter estimates converge to their true values ($\hat{\theta} \to \theta$) for all gain perturbations satisfying $\left\|\Delta(t)\right\| \le r$. □

Remark: By increasing $\lambda_{\min}(\Gamma)$, dependency of parameter estimation error bound to the initial state estimation and disturbance decreases. However, increasing Γ slows down the convergence of parameter estimate vector (10). Therefore, the trade off in selecting Γ should be considered in the design. For the case that persistent excitation condition is not met, (31) gives the resulting worst case bound on the parameter estimation error.

IV. NUMERICAL EXAMPLE

Consider the following nonlinear system:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -5 & -6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 0.2\begin{bmatrix} 0 \\ -\sin(x_2)+4u(t) \end{bmatrix} +$$

$$0.2\theta\begin{bmatrix} -\cos(x_2)+\sin(0.5t) \\ 0 \end{bmatrix} + \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix}$$

$$y = [1 \quad 0] x$$

with a unit step function input as $u(t)$, and unknown parameter $\theta = 3$ for $0 \le t < 50$, with abrupt change to $\theta = 6$ for $t \ge 50$. We select bounded continuous disturbance input signals as: $\omega_1 = \omega_2 = \sin(0.2t)e^{-0.1t}$. Moreover, consider that at time $t = 52$ the observer gain is added by values of -4, as an additive perturbation. The design parameters are chosen as $\gamma_1 = 1$, $\gamma_2 = 0.3$, $\gamma_3 = 7$, $\Gamma = 0.005$, $M = 5$, $\theta_0 = 2.8$, $\sigma_0 = 0.1$. Moreover, $\left\|\Delta(t)\right\| \le 4$ is considered as an uncertainty bound in the design. Using YALMIP toolbox as parser [20] and LMI Control Toolbox in MATLAB as solver [21], the solution is derived as: $S = [27.16 \quad 10.05]$ and $P = \begin{bmatrix} 1.42 & 0 \\ 0 & 2.50 \end{bmatrix}$, and $\varepsilon_1 = 5.0$, $\varepsilon_2 = 4.7$, $\varepsilon_3 = 5.47$, $\varepsilon_4 = 5.47$

Hence, the observer gain is obtained as $L = [16.06 \quad 4.11]^T$. For comparison purposes, we also implement the design method in [6] for the above system.

As it is shown in Fig.1, the gain obtained from the proposed resilient observer design causes the estimator to accurately track the system states while the method in [6] yields an unstable state estimation due to gain perturbation. Fig.2 shows that the parameter estimate in the proposed method also converges to its true value despite the abrupt changes of the real parameter. As we can see from the figures, when the strong gain perturbation at $t=52$ sec. occurs, the proposed design remains robust while the conventional adaptive observer [6] become unstables. As is expected, The gain perturbation does not have much effect on the estimation in the proposed method because in the observer dynamic (8), the observer gain is multiplied by output error, and since the estimation error in the proposed method converges to zero the effect of gain perturbation is omitted.

V. CONCLUSION

In this paper, we offered a systematic algorithm for designing an adaptive resilient observer for a class of nonlinear systems with containing uncertain time-varying parameters in the presence of bounded perturbation on the observer gains. The resulting LMIs can systematically obtain the robust adaptive observer gains, which ensure that state estimates under certain bound however, convergence of all the parameters, depends on the persistency of excitation.
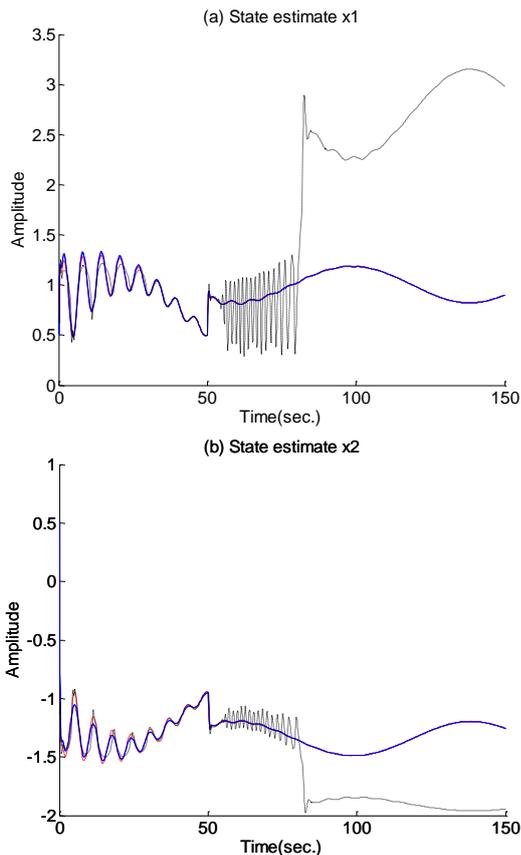
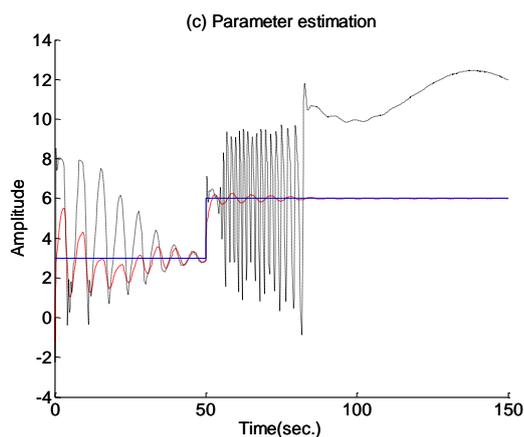Fig.1 Actual states (solid), state estimates of the proposed method (dashed), state estimates of [6] (dotted).

Fig.2 Parameter estimate using the proposed adaptive observer (dashed) vs. the method in [6] (dotted).

REFERENCES

[1] G. Bastin and M. R. Gevers, Stable adaptive observers for nonlinear time-varying systems, IEEE Trans. on Automatic Control, 33 (1988) pp. 650–658.

[2] R. Marino, Adaptive observers for single output nonlinear systems, IEEE Trans. on Automatic Control, 35 (1990) 1054–1058.

[3] R. Marino and P. Tomei, Global adaptive observers for nonlinear systems via filtered transformations, IEEE Trans. on Automatic Control, 37 (1992) 1239–1245.

[4] R. Marino, and P. Tomei, Adaptive observers with arbitrary exponential rate of convergence for nonlinear systems, IEEE Trans. Automatic Control, 40 (1995) 1300-1304.

[5] R. Rajamani and J. K. Hedrick, Adaptive observers for active automotive suspensions: Theory and experiment, IEEE Trans. on Control Systems Technology, 3 (1995) 86–93.

[6] Y. M. Cho, R. Rajamani, A systematic approach to adaptive observer synthesis for nonlinear systems, IEEE Trans. Autom. Control. 42 (1997) 534–537.

[7] R. Marino, G. L. Santosuosso, and P. Tomei, Robust adaptive observers for nonlinear systems with bounded disturbances, IEEE Trans. on Automatic Control, 46 (2001) 967–972.

[8] J. Jung, K. Hul, H.K. Fathy and J.L. Srein, Optimal robust adaptive observer design for a class of nonlinear systems via an H-infinity approach, American Control Conf., 2006, 3637-3642.

[9] M. Hasegawa, Robust adaptive observer design based on γ-Positive real problem for sensorless Induction-Motor drives, IEEE Trans. on Industrial Electronics, 53 (2006) 76-85.

[10] A. Zemouche, M. Boutayeb, G. I. Bara, Observers for a class of Lipschitz systems with extension to H_∞ performance analysis, Systems & Control Letters, 57 (2008) 18–27.

[11] C. S. Jeong, E. E. Yaz, A. Bahakeem, Y. I. Yaz, Resilient design of observers with general criteria using LMIs, Proc. of ACC, 2006, 111-116.

[12] L.H., Keel, and S.P. Bhattacharyya, Robust, fragile, or optimal?, IEEE – Trans. Autom. Control, 42 (1997) 1098–1105.

[13] P. Dorato., non-fragile contoller design: an overview, Proc. ofACC, 1998, 2829-2831.

[14] D. Famulara, C. T. Abdallah, A. Jadbabaie, P. Domto, and W. H. Haddad, Robust non-fragile LQ controllers: the statis feekback case", Proc. of ACC, 1998, 1109-1113.

[15] G.H. yang, J.L. Wang, Robust nonfragile Kalman filtering for uncertain linear systems, IEEE Trans. on Automatic Control, 46 (2001) 343-348.

[16] C. S. Jeong, E. E. Yaz, and Y. I. Yaz, Lyapunov-Based design of resilient observers for a class of nonlinear systems and general performance criteria, IEEE Multi-conference on Systems and Control, 2008, 942-947.

[17] C. S. Jeong, E. E. Yaz, and Y. I. Yaz , Stochastically resilient design of H_∞ observers for discrete-time nonlinear systems, IEEE Conf. CDC, 2007, 1227- 1232.

[18] F. Chen and W. Zhang, LMI criteria for robust chaos synchronization of a class of chaotic systems, Nonlinear Analysis, 67 (2007) 3384-3393.

[19] M. Krstic, I. Kanellakopoulos, P. Kokotovic, Nonlinear and Adaptive Control Design, John Wiley and Sons, 1995.

[20] J. Lofberg, YALMIP: A toolbox for modeling and optimization in MATLAB, IEEE Int. Symp. Computer Aided Contol Syst. Design Conf., 2004, 284-289.

[21] P. Gahinet, A. Nemirovski, A. Laub, M. Chilai, LMI control toolbox user's guide, Massachusetts: The Mathworks, 1995.

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