A New Analytical Approach for Developing an Equivalent Circuit Simulation Model for a Chip Inductor, Based on Manufacturer Specifications Given in the Datasheet

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Abstract—Inductors represent a very complex behavior especially in the field of RF techniques. Some manufacturers offer SPICE models for their inductors built up by lumped elements. The lumped element values are determined by matching the simulation model to an average of the measurements. This leads to curious numbers, and if the model is used in a simulation tool, all these values have to be adjusted if the inductor is changed. Often there are no models and measurement files available, so just the data sheet specifications are given that can be used for the work. Especially in the RF techniques several physical effects have to be taken into account and it is not sufficient to just assemble the inductor in a real circuit whose value derives from a simulation tool that works with ideal elements. The presented method shows the development of a lumped element equivalent circuit model for an inductor that is just dependent on these given datasheet specifications. The equations are tested and verified with the suggested SPICE model and the datasheet parameters of different manufacturers. The results show that the new model fits very well over the whole frequency range up to the self-resonant frequency of the element. Slightly differences of the simulated quality factors can be neglected because for the measurements the element has to be soldered on a defined printed circuit board yielding to additional losses caused by the soldering process and several coupling effects. It was shown that a simple equivalent circuit with three lumped elements results in a very good compliance of a real inductor specified by only very few parameters in the datasheet. The equations to calculate the values for the lumped elements can now be used independently from the simulation tool or mathematical software program.

Keywords—Inductor Simulation Model; QUCS; Equivalent Inductor Circuit.

I. INTRODUCTION

Nowadays there are many simulation tools on the market to calculate for example matching networks and filter circuits, most of the time based on ideal components. But just in the field of RF techniques, when different physical effects appear at high frequencies the calculated component values of the simulation tool may lead to totally different results compared to measurements performed with them assembled in a real system. In this paper a new approach was developed to find analytical equations for the desired values of the parasitic components of the equivalent circuit for such inductors, just based on the parameters specified in the datasheet. Some manufacturers offer measured s-parameter files for their inductors, others describe adopted simulation models and others do not have any special support except the datasheet parameters. The described equations also take the physical effects into account and so the model is not just an adjustment of values to achieve the same result as the measurements show, like it is done for example by Coilcraft. The given approximated values are valid only for this special part of this manufacturer and cannot be applied for an equivalent inductor of another manufacturer. With the new developed model also other inductors of different manufacturers can be easily supported, by simply applying the listed parameters to the equations. The verification and the differences based on one type of inductor are illustrated, compared to the suggested approximated equivalent circuit and the real measured values specified in the datasheet of the manufacturer, [1]. In this paper first an inductor with an inductance of 27 nH from Coilcraft was applied. Coilcraft offers both the measured data and also values for an approximated equivalent circuit, shown in Fig. 1.

![Figure 1. Equivalent circuit for an RF inductor given by Coilcraft.](image)

The value of the variable resistor $R_{var}$ is calculated by (1), where the constant $k$ relates to the skin effect and other inductor losses. All other required values can be taken from the according table. For the investigated example in this study the values given in Tab. 1 are required.

$$ R_{var} = k \cdot \sqrt{f [Hz]} $$  

(1)
These elements are determined by the overall performance of the lumped element model compared to the measurements taken of the component. Coilcraft points out that typically the self-resonant frequency of the component model will be higher than the measurement of the component mounted on the circuit board. Since data sheet specifications are based on typical production measurements, and the SPICE models are based on de-embedded measurements, the model results may be different from the data sheet specifications, [2]. The lumped element values are determined by matching the simulation model to an average of the measurements. This method results in a model that represents as closely as possible the typical frequency dependent behavior of the component up to a frequency just above the self-resonant frequency of the component.

Each manufacturer specifies a couple of values, which represent the basis for the model development. Typical values are the inductivity that is quite commonly given at a specified frequency, additionally the quality factor, a tolerance inductivity value, the self-resonance frequency of the inductor, the resistivity value if supplied with DC current and the nominal current.

II. MODEL DEVELOPMENT

In the field of RF electronics most of the time air-core coils are used, but fabrication dependent the wire is wounded around a ceramic core, to ensure that the very thin wire is protected, can be handled and the windings cannot be stretched or pressed to each other to influence the inductivity value. Between these windings and even against the environment capacitive coupling appears which leads to an increased conductance value with increasing frequency.

In principle the equivalent circuit diagram of an inductor, shown in Fig. 2 for high frequency applications, is built up by three lumped elements - the inductor with a series resistor, whose actual value is depending on frequency, current and temperature and with a capacitance in parallel, describing the previous mentioned coupling effects.

![Figure 2](image_url)

In the first step the parasitic capacitance $C_p$ is calculated by taking the self-resonant frequency $f_{res}$ and the nominal inductivity value $L_n$ from the datasheet:

$$C_p = \frac{1}{(2\pi f_{res})^2 L_n}$$  \hspace{1cm} (2)

This self-capacitance $C_p$ of the inductor together with the inductivity forms a reactance that yields bigger values with increasing frequency. Because of the fact that in the datasheet normally the inductance $L_0$ is given at a defined frequency, the inductance $L$ for the equivalent circuit can be calculated from (3), (4). In this consideration, where calculating with frequency, the series resistor is neglected and the reactive elements, which form the resonant circuit, are taken into account.

$$j \cdot \left(\omega \cdot C_p - \frac{1}{\omega L_n}\right) = -j \cdot \frac{1}{\omega L}$$  \hspace{1cm} (3)

$$L = \frac{L_n}{1 + (2\pi f_0)^2 L_n C_p}$$  \hspace{1cm} (4)

The main problem if modeling a real inductor working up to high frequencies is to consider all the different kinds of losses, that are formed by the resistance of the wire including the skin effect, the proximity effect and the dielectric losses of the self-capacitance of the inductor. If the frequency increases also the value of the resistor increases, mainly influenced by the skin-effect. The mathematical correlation between the time dependent current change and the radial current distribution in the solid wire of the inductor can be expressed by the differential equation for the BESSEL function of the zero order given in (5), (6). A detailed derivation is given in [3].

$$\frac{d^2E}{dr^2} + \frac{1}{r} \frac{dE}{dr} + k^2 \cdot E(r) = 0$$  \hspace{1cm} (5)

$$k^2 = -j \cdot \omega \cdot \sigma \cdot \mu.$$  \hspace{1cm} (6)

In such inductors only solid wires are used, which show different cross-sections for changing current loads. For such a simple geometry it is possible to approximately calculate the influence of the current displacement and even the skin depth of the current in the wire, if assuming that the current flow is distributed constantly across its cross-section. The alternating current resistance $x$ of a solid wire with radius $r_0$ emanating from (5) is calculated by

$$x = \frac{r_0}{2} \cdot \frac{1}{\sqrt{\pi \cdot f \cdot \sigma \cdot \mu}}.$$

Before it is possible to find the solution with the help of a series expansion of the Bessel function of the first kind, it is necessary to estimate the radius of the wire with which the inductor is built up. In the datasheet only two specifications are given that can be applied for computing the wire diameter – the

TABLE I. PARAMETER VALUES FOR THE LUMPED ELEMENTS OF THE EQUIVALENT CIRCUIT FOR A 27 NH INDUCTOR FROM COILCRAFT.

<table>
<thead>
<tr>
<th>Part number</th>
<th>R1 [Ω]</th>
<th>R2 [Ω]</th>
<th>C [pF]</th>
<th>L [nH]</th>
<th>k [Ω/Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0402CS-27N</td>
<td>4</td>
<td>0.001</td>
<td>0.1</td>
<td>26</td>
<td>8.00E-05</td>
</tr>
</tbody>
</table>


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ohmic resistance of the wire and the nominal current. For developing this model the current was taken into account. The nominal current value defines a specified temperature rise of the wire in a certain time, given in [°C/s]. The temperature of a component depends on the thermodynamic (heat transfer) characteristics of the component, the circuit board, the solder connection, the surrounding environment, the impedance of the component, and the current through the component. The power dissipation of a component is a function of all of these variables, [4]. Coilcraft specifies different current ratings depending on the type of inductor. In the case of chip inductors the rms current (\(I_{\text{rms}}\)) is given, which represents the root mean square current that causes the temperature of the component to rise by 15°C above ambient at 25°C. The manufacturer uses a pulsed waveform with 70% duty cycle, meaning that the rate can be calculated by dividing the 15°C by 0.7s. This rate was taken to calculate the cross-section area of the copper wire by solving (9) and the wire radius was computed sequentially employing (10, 11). Furthermore the following physical parameters for the electro-copper are taken from [5, 6] and depicted in Tab. 2.

### TABLE II. PHYSICAL PROPERTIES OF THE ELECTRO-COPPER USED FOR THE SIMULATION MODEL.

<table>
<thead>
<tr>
<th>Property</th>
<th>Unity</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass density at 20°C</td>
<td>8.94 kg/dm³</td>
</tr>
<tr>
<td>specific thermal capacity at 20°C</td>
<td>0.386 J/(g*K)</td>
</tr>
<tr>
<td>specific electrical conductivity</td>
<td>58 m/(Ω*mm²)</td>
</tr>
</tbody>
</table>

Equation (8) has to be changed to calculate the cross section area by

\[
\left(\frac{\Delta T}{T}\right) = \frac{I_{\text{rms}}^2}{P_{\text{Cu}} \cdot \sigma_{\text{Cu}} \cdot \epsilon_{\text{Cu}} \cdot A^2} \tag{8}
\]

Considering

\[
A = r_0^2 \cdot \pi \tag{9}
\]

and applying the numbers given in Tab. 2 the radius finally results in

\[
r_0 = 0.15 \cdot \sqrt[4]{\frac{I_{\text{rms}}^2}{\epsilon_{\text{Cu}}}} \tag{11}
\]

As mentioned in [3, 7] the rule of thumb for \(x > 1\) related to the solution of the Bessel function of the first kind for big numbers leads to

\[
R_S = R_{\text{DC}} \cdot \left(x + \frac{1}{4} + \frac{3}{64 \pi} \right). \tag{12}
\]

If in the last step all equations are combined, the frequency dependent resistor value can be expressed by (13), if applying \(\mu_0 = 4\pi \cdot 10^{-7}\) and for \(\sigma\) refer to Tab. 2.

\[
R_S = R_{\text{DC}} \cdot \left(0.001135 \cdot \sqrt[4]{\frac{I_{\text{rms}}^2}{\mu_0}} + 0.25 + \frac{41.3}{\sqrt{\pi} \cdot \sqrt[4]{\frac{I_{\text{rms}}}{\mu_0}}} \right) \tag{13}
\]

At this point all three elements of the equivalent circuit of the inductor can be described analytically, just dependent on known parameters specified in the datasheet of a manufacturer. The resistance value can be calculated by (13), for the inductor value (4) is applied and the parallel capacitance value derives from (2).

### III. VERIFICATION OF THE SIMULATION MODEL

As it was already mentioned Coilcraft offers both, the reconstructed simulation model built up by lumped elements and the \(s\)-parameter file of the measurement of the inductor. The verification of the new developed model was done with QUCS, [8]. First of all the two circuits are drawn in the schematic editor. As long as the elements are constant in value they are implemented as discrete devices, while for the frequency dependent elements the equation defined two-port RF block is used. The type of them is changed to ‘A’ and in every case \(P11 = P22 = 1\), \(P21 = 0\) and for \(P12\) the specific equation is applied. The following Fig. 3 shows the different circuits, the lumped element circuit suggested by the manufacturer and the equivalent circuit consisting of only three lumped elements, as developed in this paper.

Figure 3. Two different kinds of simulation models are shown – manufacturer suggested equivalent circuit and the new developed equivalent circuit.
Additional to the schematics it is necessary to assign the values given in the datasheet to the variables through the equation block. At last the frequency range for the simulation is defined and the simulation is started. The simulation was done for an inductor of Coilcraft and the model was also tested for two further manufacturers – Würth Elektronik and Johanson Technology.

Fig. 4 shows the diagram of the measured quality factor from the Coilcraft inductor 0402CS-27NX_L_. Additionally the typical values for the quality factor of 49 @ 900 MHz and 63 @ 1.7 GHz were given in the datasheet. Fig. 5 shows the results for both models – the dashed line reflects the given equivalent circuit of Fig. 1 and the solid curve depicts the result of the new developed model.

The same simulations are done for a 27 nH inductor from Würth Elektronik of the type 744765127A. Fig. 6 shows the measured data from the manufacturer, while Fig. 7 indicates the result of the simulation model from Fig. 3 but with the changed datasheet values. In the datasheet a quality factor of 26 @ 250 MHz and 48 @ 900 MHz is mentioned.
At last an inductor of a third manufacturer named Johanson Technology was tested. The type number is L-07C27NJV6T. The manufacturer offers an own software where the measurement of the different inductors are stored. Therefore it is possible to verify exactly the values that are achieved in the simulation model with QUCS at the given marker positions. Fig. 8 shows the measured data from Johanson Technology and Fig. 9 indicates the result of the simulation model. Three quality factor values are verified, which are 9 @ 99 MHz, 13.8 @ 246 MHz and 18.2 @ 899 MHz.

![Figure 8. Measured quality factors from Johanson Technology software.](image)

![Figure 9. Simulation result with the new model applied to the given data of Johanson Technology.](image)

IV. Summary

In this paper a new approach for developing an equivalent circuit for inductors is presented. The shown equations are valid to generate models independently from the manufacturer and are not just an approximation of a measurement file. Several physical effects are considered and therefore the model can be applied up to the self-resonant frequency of the component itself. The model is built up by only three lumped elements and the mathematical description of their values allows for employing them in other mathematic software tools. The following Tab. 3 depicts the resulting quality values computed with the simulation tool compared to the values given by the manufacturer, which quite commonly are derived as a mean value of several measurements.

<table>
<thead>
<tr>
<th>Manufacturer</th>
<th>Frequency [MHz]</th>
<th>Q_{MODEL}</th>
<th>Q_{MEASUREMENT}</th>
<th>Diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coilcraft</td>
<td>0402CS-27NX_L</td>
<td>900</td>
<td>58.7</td>
<td>49</td>
</tr>
<tr>
<td>Würth Elektronik</td>
<td>744765127A</td>
<td>1700</td>
<td>61.6</td>
<td>63</td>
</tr>
<tr>
<td>Johanson Technology</td>
<td>L-07C27NJV6T</td>
<td>250</td>
<td>25.3</td>
<td>26</td>
</tr>
<tr>
<td>L-07C27NJV6T</td>
<td>900</td>
<td>43</td>
<td>48</td>
<td>9</td>
</tr>
<tr>
<td>07C27NJV6T</td>
<td>99</td>
<td>12.1</td>
<td>13.8</td>
<td>9</td>
</tr>
<tr>
<td>899</td>
<td>19.5</td>
<td>18.2</td>
<td>1.3</td>
<td></td>
</tr>
</tbody>
</table>

Referring to Tab. 3 it can be seen, that the model fits better for inductors with lower quality factors than with higher ones. This means that if implementing an inductor as an example for a matching network it is not sufficient to just look for the right value of the component, it is also necessary to consider the physical effects and the inductor assembly that influences the quality factor. This is also necessary in the case of defining a second source manufacturer for an inductor component of the RF circuit. Summing up this new developed equivalent circuit offers a good estimation how the inductor will act in a real circuit at high frequencies.

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