Partial Discharge Localization in Stator Winding of Generators using R, L, C Ladder Network

S.M.H. Hosseini 1, S.M. Hosseini 2, H.R. Khezri 3

1, 2 Islamic Azad University, South Tehran Branch (IAU), Tehran, Iran
3 Islamic Azad University, Bafgh Branch (IAU), Yazd, Iran

Abstract- Partial discharge (PD) is a major source of insulation failure in power electrical machines. The location of a PD source is of crucial importance in both the maintenance and repair of an electrical machine. This paper applies the knowledge that the poles of a PD current frequency spectrum do not change, whilst the zeros vary with the position of a PD source within the winding. An algorithm based on this approach has been developed for the localization of PDs. The algorithm adopts the well-known ladder network to model the electrical machine winding, and then estimates the parameters of this model from the poles of the PD current frequency spectrum. This provides the necessary information to be able to calculate a PD signal from different source locations within the model. Finally, the position of a PD source can then be estimated by a comparison of the measured and calculated PD signals. Simulation and experimentation results are demonstrated in the paper.

Keywords- Generators; Partial Discharge; RLC Ladder Network; Spectrum

I. INTRODUCTION

The generators are one of the most important and expensive equipment in power systems [1]. Once a generator is damaged, replacement costs of a large HV generator might reach up to a few million pounds in the UK. If an incipient failure of a generator is detected before it leads to a catastrophic failure, the generator may be repaired on site or replaced according to a scheduled arrangement [2]. Therefore, conditions of critical assets, i.e. generator, for utilities should be closely and continuously monitored in order to ensure maximum uptime [3]. The so-called condition-based maintenance may reduce risks of forced outages and damages to adjacent equipment. Generators interruptions in service and failures usually result from dielectric breakdown, winding distortion caused by short circuit withstand, winding and magnetic circuit hot spot, etc. Winding distortion faults may cause catastrophic failures of generators such as dielectric breakdown and short circuit [4]. Most of the time, the greatest problem is the internal resonance which occurs when the frequency of the input surge is equal to some of the resonance frequencies of the generator. This overvoltage's are characterized by a very short rise time [5]. The experience shows that VFTOs within generator can be expected to have even a rise time of 0.1 μs and amplitude of 2.5 p.u. Most of the time, resonant overvoltage's can cause a flashover from the windings to the core or between the turns [6]. The inter-turn insulation is particularly vulnerable to high-frequency oscillation and therefore the study of the distribution of inter-turn over voltages is of essential interest. The VFTOs produced by voltage source in generator depend not only on the connection between the voltage source and generator, but also on the generator parameters and type of the winding. Different models of generator windings have been suggested for transient studies. The most of them have been approved by different researchers [7]. The generator engineers use this model to predict the surge voltage distribution along the generator winding [8]. Generator modeling methods can be classified to Gray Box or parametric modeling and Black Box models. The Gray Box models can be used by designers to study the resonance behavior of generator winding and the distribution of electrical stresses along the generator windings. As a result, in the design stage, the Gray Box model has privileges to the Black Box model. The Gray Box models can be categorized as: "RLC Ladder Network Model" and "MTL Model" [6]. The fundamental elements of the Ladder Network model are the lumped R, L and C elements. The frequency limitation for the validity of this model is in the range of a few hundred kHz. In order to extend this range to a few MHz, it is necessary to use a turn-to-turn modeling procedure instead of disk-to-disk modeling. This procedure will result in a large scale system, which would be difficult to simulate and to analyze such a sophisticated system [7]. The other solution to this problem is the application of the hybrid model which can be built by a combination of Gray and Black Box models. In this model, due to application of a Black Box approach, the order of the network is reduced substantially. However there is no transient voltage distribution information available along the winding in a Black Box model. To overcome this problem a method will be introduced in this paper which is based on the Multi-conductor Transmission Line (MTL) theory. Using this theory, the number of equations and the size of the memory required for the calculation decreased significantly.

In addition, due to the application of the distributed parameters, the model accuracy will be expanded over MHz frequency range [5] [8]. The published works on frequency dependent modeling of transformer is more focused on the RLC Ladder Network model in past. While the published works on MTL
modeling is mostly concentrated on modeling of electrical rotating machines and also on only the homogenous transformer windings ignoring the frequency dependency of the winding insulation parameters. The present paper deals with the problem of evaluation of fast transient voltages in distribution generator windings. Therefore a refined generator model is presented, based RLC Ladder Network theory [2, 3].

II. BACKGROUND THEORY

A PD generated within an electrical machine winding can have an associated frequency range between a few kHz to hundreds MHz. Over this range a ladder network model as in Figure 1 may be employed.

![Diagram](image)

Figure 1. A ladder network model for a electrical machine winding, PD at Node 1

Note that mutual inductances have not been explicitly depicted in Figure 1. The PD occurrence can be simulated as a current pulse injected into the network. To obtain the output current through the neutral point in such a circuit, it is very convenient to use state-space equations as follows:

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= C_1 x + C_2 x^*
\end{align*}
\]

With

\[
\begin{align*}
x &= [i_1(t), i_2(t), ..., i_N(t), v_1(t), v_2(t), ..., v_N(t)]^T \\
A &= \begin{bmatrix} -L^{-1}R & -L^{-1}N \\ \Theta^{-1}M & \Theta^{-1}G \end{bmatrix} \\
B &= \begin{bmatrix} 0 \\ \Theta^{-1}K \end{bmatrix} \\
C_1 &= [1, 0, ..., 0, -G_s, 0, ..., 0]; \\
C_2 &= [0, 0, ..., 0, -G_s, 0, ..., 0]; \\
K &= [0, 0, ..., 0, 1_r, 0, ..., 0];
\end{align*}
\]

Note that the position of 1 in vector K is decided by the input position.

\[
\begin{align*}
L, \Theta \text{ and } G \text{ are the inductance, capacitance and admittance matrices of this system. They are given by:}
L &= \begin{bmatrix}
L & M_{12} & M_{13} & \cdots & \cdots & M_{1N} \\
M_{12} & L & M_{12} & \cdots & \cdots & \vdots \\
M_{13} & M_{12} & L & \cdots & \cdots & \vdots \\
\vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\
M_{1N} & \cdots & \cdots & \cdots & L & M_{12} \\
\end{bmatrix} \\
\Theta &= \begin{bmatrix}
(C_s + 2C_g) & -C_s & 0 & \cdots & 0 \\
-C_s & (C_s + 2C_g) & \cdots & \cdots & \cdots \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
0 & \cdots & -C_s & C_s + C_g & \cdots \\
\end{bmatrix} \\
G &= \begin{bmatrix}
-G_s & G_s & 0 & \cdots & 0 \\
G_s & -(2G_s + G_g) & \cdots & \cdots & \cdots \\
0 & \cdots & G_s & -(G_s + G_g) & \cdots \\
\vdots & \vdots & \vdots & \ddots & \ddots \\
0 & \cdots & \cdots & G_s & -(G_s + G_g) \\
\end{bmatrix}
\end{align*}
\]

Note that C_b is the bushing capacitance.

The frequency response can be calculated by taking the Laplace Transform of Eq. (1).

\[
\frac{Y(s)}{U(s)} = (C_1 + C_2 s) (sI - A)^{-1} B
\]

(2)

Since the PD current source is assumed to have a unitary distribution, the frequency spectrum of the detected PD can be considered as the frequency response of the system and Eq. 2 may be written as:

\[
\frac{Y(s)}{U(s)} \approx 1
\]

(3)

In Eq. 3 only the matrix B is related to the position of the input (PD current source location). It can be seen that the matrix A is not affected by the position of the PD current source. Also, the occurrence of a discharge does not affect the system structure. Therefore, the poles of the system, which are the eigenvalues of the matrix A, are completely determined by the system itself. Only the zeros contain information related to the PD site-of-origin. In another words, the poles of the frequency spectrum of any detected PD current signal may be used for system identification (parameters \(L_s, C_s, C_g\) and \(r\) of the model), whereas the zeros of the transfer function may be used for PD localization.
III. PARAMETER ESTIMATION FROM DETECTED PD CURRENT

Based on the theory discussed above, the procedure for estimating electrical machine parameters from detected PD current is composed of three steps. These are as follows:

1) Obtain the frequency spectrum $Y(s)$ of the terminal PD current signal.
2) Estimate the poles of the system from the calculated frequency spectrum.
3) Estimate the parameters in the model, as depicted in Figure 1, from the estimated poles.

Step 1 can be achieved easily through the use of a Fast Fourier Transform (FFT). In the second step, the frequency spectrum, which is the frequency response of the system, is modeled by a transfer function [10] as follows:

$$ Y(S) = \frac{\beta(S)}{\alpha(S)} $$

(4)

Where $\alpha(S)$ and $\beta(S)$ are polynomials and defined by

$$ \alpha(S) = \alpha_n S^n + \alpha_{n-1} S^{n-1} + ... + \alpha_1 S + \alpha_0 $$

(5)

$$ \beta(S) = b_m S^m + b_{m-1} S^{m-1} + ... + b_1 S + b_0 $$

(6)

Substituting Eq.5 and Eq.6 into Eq.4, the following equation is obtained:

$$ Y(S) = X(S) \times \theta $$

(7)

Where

$$ X(S) = [-S^m Y(S) \ldots -SY(S) S^m \ldots S 1] $$

(8)

And

$$ \theta = [\alpha_n \ldots \alpha_1 \ b_m \ldots b_1 \ b_0] $$

(9)

Then the coefficients can be obtained through applying a least square optimization. After that, it is not difficult to get the poles of the system, which are the roots of polynomial $\alpha(S)$.

Step 3 is a nonlinear optimization problem. The objective function is to minimize the difference between the eigenvalues of matrix A and the calculated poles. The eigenvalues and poles are sorted by frequency in descending order.

$$ \text{eig}(A) = \text{eig} \begin{bmatrix} -L^{-1}R & -L^{-1}N \\ \Theta^+M & \Theta^+G \end{bmatrix} = \text{poles} = \text{roots}(\alpha(S)) $$

(10)

There are a wide variety of non-linear optimization algorithms available. This paper has tried nonlinear least squares (NONLSQ) and a genetic algorithm (GA) approach. It has found that with the high number of parameters and the nature of the problem, several local minimums appear. For these reasons, NONLSQ has difficulties and often does not converge to the global minimum point. However GA, with its evolution based survival of the fittest approach to parameter optimization, provides quite good results. More details on genetic algorithms can be found in the reference [11].

A. Calculation of model parameters

RLC model parameters consist of capacitance, inductance, resistance and conductance matrices. They depend on conductors and insulations geometry and characteristics, geometrical dimensions of the generator, winding type and position of each winding [10, 11].

IV. CAPACITANCE MATRIX

There are various capacitances between different conductors. The capacitance between two adjacent turns in a coil can be calculated by assuming parallel plate capacitor approximation as:

$$ C_T = \frac{\varepsilon_r \varepsilon_0 \times \pi D_m (w + t_p)}{t_p} $$

(11)

Where $D_m$ is the winding diameter, $w$ is the width of the conductor in axis direction, $t_p$ is paper thickness in both sides of the conductor, $\varepsilon_0 = 8.85 \times 10^{-12} \text{F/m}$ and $\varepsilon_r$ is the relative permittivity of paper.

V. INDUCTANCE MATRIX

In high frequencies it can be assumed that the penetration of magnetic flux into the laminated iron core of generator is neglected, so the winding can be regarded as a conductor in free space surrounded by insulation. $n \times n$ inductance matrix is formed by self and mutual inductances between different turns of the winding. Mutual inductance between two circular filaments is calculated using the formula developed by Maxwell is obtained:
\[
\begin{align*}
L_{AB} &= \frac{2\mu}{K} \sqrt{R_A R_B} \left[ \left( 1 - \frac{K^2}{2} \right) K(k) - E(k) \right] \\
k &= \frac{4R_A R_B}{\sqrt{(R_A + R_B)^2 + S^2}}
\end{align*}
\] (12)

Where \( R_1 \) and \( R_2 \) is the radius of the circular filaments 1 and 2, \( S \) is the distance between circular filaments, \( \mu_0 \) is the permeability of free space and \( K(k) \) and \( E(k) \) are the complete elliptic integrals of the first and second kind. Inductance calculations are based on geometric entities. The formula developed to compute the self-inductance (in Henry) is as follows:

\[
L_{AA} = \mu_0 a \left[ \frac{1}{2} \left( 1 + \frac{3}{2a} \right) LN \left( \frac{8}{C/2a} \right) - 0.84834 + 0.2041 \left( \frac{C}{2a} \right)^2 \right]
\] (13)

Where \( \mu_0 \) is the permeability of free space, a geometric mean radius of each coil and \( C \) is the length of the winding cross section is square.

### VI. SERIES RESISTANCES

The per unit length resistance of conductor can be obtained by (12):

\[
R = \frac{1}{2(h+w)} \sqrt{\frac{\pi F \mu}{\delta}}
\] (14)

Where \( \delta \) is copper conductivity all, \( F \) is frequency and terms have been defined before. Other dimensions have been shown in Fig. 4.

### VI. SERIES RESISTANCES

The per unit length resistance of conductor can be obtained by (12):

\[
R = \frac{1}{2(h+w)} \sqrt{\frac{\pi F \mu}{\delta}}
\] (14)

Where \( \delta \) is copper conductivity all, \( F \) is frequency and terms have been defined before. Other dimensions have been shown in Fig. 4.

### VII. PARALLEL CONDUCTANCES

Parallel conductances are due to dielectric losses and can be obtained by:

\[
\left[ G \right] = 2\pi F \left[ C \right] \tan \delta
\] (15)

The parameters of RLC model are determined based on numerical field analysis methods (e.g. finite element method), by using maxwell software.

### Table I. Parameters of the Machine Under Test

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated voltage</td>
<td>6000</td>
<td>v</td>
</tr>
<tr>
<td>Rated power</td>
<td>250</td>
<td>kw</td>
</tr>
<tr>
<td>Rated speed</td>
<td>1500</td>
<td>r.p.m</td>
</tr>
<tr>
<td>Rated frequency</td>
<td>50</td>
<td>Hz</td>
</tr>
<tr>
<td>Winding Connection</td>
<td>star</td>
<td></td>
</tr>
<tr>
<td>Number of turns per phase</td>
<td>176</td>
<td></td>
</tr>
<tr>
<td>Number of coils per phase</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>Number of turns in a coil</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>Conductor dimensions</td>
<td>11.5*236</td>
<td>mm</td>
</tr>
</tbody>
</table>

### Table II. Parameters of the Oil Under Test

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>L(mH)</td>
<td>Real</td>
<td>17.2</td>
</tr>
<tr>
<td></td>
<td>Estimated</td>
<td>17.7</td>
</tr>
</tbody>
</table>

The calculated current signals are then fed into a software implementation of the algorithm for PD localization as previously described. The first PD current used is that with the source located at the 2nd section node away from the neutral point. Figure 8, 9 and 10 is a comparison between the...
measured current spectrum and the estimated current spectrum.

![Schematic machine with the coil under test](image1)

**Figure 7.** Schematic machine with the coil under test

![Comparison between computed and measured current spectra](image2)

**Figure 8.** The comparison between the current spectrum of the simulated and measured, when PD be injected at node 1.

![Comparison between computed and measured current spectra](image3)

**Figure 9.** The comparison between the current spectrum of the simulated and measured, when PD be injected at node 6.

![Comparison between computed and measured current spectra](image4)

**Figure 10.** The comparison between the current spectrum of the simulated and measured, when PD be injected at node 10.

<table>
<thead>
<tr>
<th>PD&lt;sub&gt;i&lt;/sub&gt;</th>
<th>Real position</th>
<th>Estimated position</th>
</tr>
</thead>
<tbody>
<tr>
<td>PD&lt;sub&gt;1&lt;/sub&gt;</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>PD&lt;sub&gt;2&lt;/sub&gt;</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>PD&lt;sub&gt;3&lt;/sub&gt;</td>
<td>10</td>
<td>11</td>
</tr>
</tbody>
</table>

**VIII. CONCLUSION**

An algorithm for electrical machine partial discharge localization has been developed in this paper. The approach is based upon frequency spectrum analysis of PD current signals. The algorithm initially estimates the parameters of the electrical machine from the poles of the discharge frequency spectrum. The position of a PD source can then be deduced by calculating the PD current waveforms with the source simulated at various locations within the electrical machine. These waveforms are then compared to the observed waveform to determine the closest match. This algorithm does not require any predetermined information about the electrical machine. The PD site-of-origin is determined purely from a measured PD current. Simulation and experimentation both show promising results.

**REFERENCES**


S. M. Hassan Hosseini was born in Tehran, in 1969. He received his B.Sc. degree in electrical power engineering from Mashhad Ferdowsi University, Mashhad, Iran in 1993. He received his M.Sc. and Ph.D. degrees in electrical power engineering in 2000 and 2005 from Azad University South-Tehran Branch and Science & Research Branch, Tehran, Iran, respectively. He held the position of Assistant Professor in Azad University South-Tehran Branch from 2005. From 2008 to 2009 he was as deputy and from 2009 till 2011 he was as the manager of electrical engineering department of Azad University South-Tehran Branch. His research interest is transient modeling of transformer, Partial Discharge, High Voltage Engineering, Electrical Insulation, Substation and Hydropower.

H.R. Khezri was born in Bafgh, Iran, in 1980. He received the B.Sc. degree in electrical power engineering in 2011 from Islamic Azad University of Abarkooh, Iran, and the M.Sc. degrees in electrical power engineering in 2013, from the Islamic Azad University, Tehran, Iran. Since 2011, He joined as a lecturer of Electrical Engineering Department, Faculty of Engineering, and Islamic Azad University Bafgh Branch. His research interest is transient modeling of transformers and Generators.

S. M. Hosseini Bafghi was born in Bafgh, Iran, in 1986. He received the B.Sc. degree in electrical power engineering in 2009 from Islamic Azad University of Mehriz, Iran, and the M.Sc. degrees in electrical power engineering in 2013, from the Islamic Azad University, Tehran, Iran. Since 2011, He joined as a lecturer of Electrical Engineering Department, Faculty of Engineering, and Islamic Azad University Bafgh Branch. His research interest is transient modeling of transformers and Generators.