Determination and Analysis of Transmission Line Performance (Short-Medium Transmission Line)

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Abstract-In power system we need to transference power from one side that is name sending end to other side that its name is receiving end and this transference can be done by cables. These parts from the network called transmission lines and classified as three regions (short, medium and long). In this paper we explain two regions with mathematical analysis and comparison results.

Keywords- Short transmission line, Medium transmission line, Power systems

I. INTRODUCTION

Transmission line is the long conductor with special design (bundled) to carry bulk amount of generated power at very high voltage from one station to another as per variation of the voltage level.

When transportation this generated power at long distance because high current a lot of losses and heat appear in this conductor this cause unstable and faults in the power system network so can connect rising transformer voltage in send end in receiving end connect down transformer voltage at inside desired.

In transmission line determination of voltage drop, transmission efficiency, line loss ... etc. are important things to design. These values are affected by line parameter R, L and C of the transmission line.

Length wise transmission lines are three types:

A. Short transmission line
   • Length is about 80 km.
   • Voltage level is up to 20 kV
   • Capacitance effect is negligible
   • Only resistance and inductance are taken in calculation capacitance is neglected

B. Medium transmission line
   • Length is about 80 km to 250 km
   • Operational voltage level is from 20 kV to 100 kV
   • Capacitance effect is present
   • Distributed capacitance form is used for calculation purpose.

C. Long transmission line
   • Length is 250 and longer km
   • Voltage level is above 100 kV
   • Line constants are considered as distributed over the length of the line.

This paper is devoted two types, short and medium transmission lines only and mathematical analyses about them. As short transmission line in second section [1], medium transmission line and methods in 3rd section [1]-[2], and then solving example to compare the result in section 4th.[4]

II. SHORT TRANSMISSION LINE

The transmission lines which have length about 80 km are generally referred as short transmission lines. For short length, the shunt capacitance of this type of line is neglected and other parameters like electrical resistance and inductor of these short lines are lumped, hence the equivalent circuit is represented is given below. Let’s draw the vector diagram for this equivalent circuit, taking receiving end current Ir as reference. The sending end and receiving end voltages make angle with that reference receiving end current, of φs and φr, respectively.

Figure 1. Short transmission line representation
As the shunt capacitance of the line is neglected, hence sending end current and receiving end current is same, i.e. \( I_s = I_r \). Now if we observe the vector diagram carefully, we will get, \( V_s \) is approximately equal to:

\[
V_S = I_r . R \cos \phi_r + I_r . X \sin \phi_r
\]

That means,

\[
V_s \cong V_r + I_r . R \cos \phi_r + I_r . X \sin \phi_r
\]

It is assumed that \( \phi_s \cong \phi_r \) As there is no capacitance, during no load condition the current through the line is considered as zero, hence at no load condition, receiving end voltage is the same as sending end voltage. As per definition of voltage regulation of power transmission line,

\[
\% \text{ regulation} = \frac{(V_s - V_r)}{V_r} \times 100\%
\]

\[
= \frac{(I_r . R \cos \phi_r + I_r . X \sin \phi_r)}{V_r} \times 100\%
\]

Per unit regulation = \( V_r \cos \phi_r + V_x \sin \phi_r \)

Here, \( v_r \) and \( v_x \) are per unit resistance and reactance of the short transmission line respectively.

Any electrical network generally has two input terminals and two output terminals. If we consider any complex electrical network in a black box, it will have two input terminals and output terminals. This network is called two - port network. Two port model of a network simplifies the network solving technique. Mathematically a two port network can be solved by 2 by 2 matrix. A transmission as it is also an electrical network; line can be represented as two port network. Hence two port network of transmission line can be represented as 2 by 2 matrix.

Voltage and currents of the network can represented as,

\[
V_s = A \cdot V_r + B \cdot I_r \quad (1)
\]

\[
I_s = C \cdot V_r + D \cdot I_r \quad (2)
\]

Where, \( A, B, C \) and \( D \) are different constant of the network.

If we put \( I_r = 0 \) at equation (1), we get:

\[
A = \frac{V_s}{V_r} \quad (3)
\]

Hence, \( A \) is the voltage impressed at the sending end per volt at the receiving end when receiving end is open. It is dimension less. If we put \( V_r = 0 \) at equation (1), we get:

\[
B = \frac{V_s}{I_r} \quad (4)
\]

That indicates it is impedance of the transmission line when the receiving terminals are short circuited. This parameter is referred as transfer impedance.

\[
C = \frac{I_s}{V_r} \quad (5)
\]

\( C \) is the current in amperes into the sending end per volt on open circuited receiving end. It has the dimension of admittance.

\[
D = \frac{I_s}{I_r} \quad (6)
\]

\( D \) is the current in amperes into the sending end per amp on short circuited receiving end. It is dimensionless. Now from equivalent circuit, it is found that,

\[
V_s = V_r + I_r Z \quad \text{and} \quad I_s = I_r
\]

Comparing these equations with equation 1 and 2

We get,

\[
A = 1, B = Z, C = 0 \text{ and } D = 1.
\]

As we know that the constant \( A, B, C \) and \( D \) are related for passive network as,

\[
AD - BC = 1.
\]

Here,

\[
A = 1, B = Z, C = 0 \text{ and } D = 1
\]

\[
\Rightarrow 1.1 - Z . 0 = 1
\]

So the values calculated are correct for short transmission line.

From above equation (1),

\[
V_s = A V_r + B I_r
\]

When \( I_r = 0 \) that means receiving end terminals is open circuited and then from the equation 1, we get receiving end voltage at no load

\[
V_r' = \frac{V_s}{A}
\]

And as per definition of voltage regulation of power transmission line,

\[
\% \text{ V reg.} = \frac{(V_s - V_r)}{V_r} \times 100\%
\]

The efficiency of short line as simple as efficiency equation of any other electrical equipment, that means

\[
\% \text{ Efficiency (\( \eta \))} = \frac{\text{Power at received end}}{\text{Power at sending end}} \times 100\%
\]

\[
= \frac{P \text{ rec. end}}{(P \text{ rec. end} + 3I_r^2 R)} \times 100\%
\]

\( R \) is per phase electrical resistance of the transmission line.

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\]

\( R \) is per phase electrical resistance of the transmission line.
III. MEDIUM TRANSMISSION LINE

The transmission line having its effective length more than 80 km but less than 250 km is generally referred to as a medium transmission line. Due to the line length being considerably high, admittance \( Y \) of the network does play a role in calculating the effective circuit parameters, unlike in the case of short transmission lines. For this reason the modeling of a medium length transmission line is done using lumped shunt admittance along with the lumped impedance in series to the circuit. These lumped parameters of a medium length transmission line can be represented using three different models, namely:

1. Nominal \( \Pi \) representation
2. Nominal \( T \) representation
3. End Condenser Method

Let’s now go into the detailed discussion of these above mentioned models.

A. Nominal \( \Pi \) representation

In case of a nominal \( \Pi \) representation, the lumped series impedance is placed at the middle of the circuit whereas the shunt admittances are at the ends. As we can see from the diagram of the \( \Pi \) network below, the total lumped shunt admittance is divided into 2 equal halves, and each half with value \( Y/2 \) placed at both the sending and the receiving ends while the entire circuit impedance is between the two. The shape of the circuit so formed resembles that of a symbol \( \Pi \), and for this reason it is known as the nominal \( \Pi \) representation of a medium transmission line. It is mainly used for determining the general circuit parameters and performing load flow analysis.

As we can see here, \( V_S \) and \( V_R \) is the supply and receiving end voltages respectively, and \( I_S \) is the current flowing through the supply end. \( I_R \) is the current flowing through the receiving end of the circuit. \( I_1 \) and \( I_3 \) are the values of currents flowing through the admittances. And \( I_2 \) is the current through the impedance \( Z \).

Now applying KCL, at node P, we get:

\[
I_S = I_1 + I_2 \tag{3}
\]

Similarly applying KCL, to node Q:

\[
I_2 = I_3 + I_R \tag{4}
\]

substituting equation (4) to equation (3)

\[
I_S = I_1 + I_2 = (y/2) \cdot V_S + (y/2) \cdot V_R + I_R \tag{5}
\]

Now by applying KVL to the circuit,

\[
V_S = V_R + Z \cdot I_2 = V_R + Z \cdot \left[ \left( \frac{y}{2} \right) V_R + I_R \right]
\]

\[
= \left( Z \left( \frac{y}{2} \right) + 1 \right) V_R + Z I_R \tag{6}
\]

Now substituting equation (6) to equation(5) we get:

\[
I_S = y \left( \frac{y}{4} \right) Z + 1 \cdot V_R + \left( \frac{y}{2} \right) Z + 1 I_R \tag{7}
\]

Comparing equation (6) and (7) with the standard ABCD parameter equations

\[
\begin{align*}
V_s &= A V_r + B I_r \\
I_s &= C V_r + D I_r
\end{align*}
\]

We derive the parameters of a medium transmission line as:

\[
\begin{align*}
A &= (y/2) Z + 1 \\
B &= Z \omega \\
C &= y \left( \frac{y}{4} \right) Z + 1 \\
D &= (y/2) Z + 1
\end{align*}
\]

B. Nominal \( T \) representation

In the nominal \( T \) model of a medium transmission line the lumped shunt admittance is placed in the middle, while the net series impedance is divided into two equal halves and placed on either side of the shunt admittance. The circuit so formed resembles the symbol of a capital \( T \), and hence is known as the nominal \( T \) network of a medium length transmission line and is shown in the diagram below.
Here also \( V_s \) and \( V_r \) is the supply and receiving end voltages respectively, and \( I_s \) is the current flowing through the supply end. \( I_r \) is the current flowing through the receiving end of the circuit. Let \( M \) be a node at the midpoint of the circuit, and the drop at \( M \), be given by \( V_m \). Applying KVL to the above network we get,

\[
\frac{V_s - V_m}{Z/2} = y \frac{V_m - V_r}{Z/2}
\]

or:

\[
V_m = \left[ \frac{2(V_s + V_r)}{yZ + 4} \right]
\]  
\( \text{(8)} \)

And the receiving end current:

\[
I_r = \left[ \frac{2(V_m - V_r)}{Z/2} \right]
\]  
\( \text{(9)} \)

Now sub. \( V_m \) from equation \( (8) \) to \( (9) \) we get:

\[
I_r = \frac{[(2V_s + V_r) / yZ + 4] - V_r}{Z/2}
\]

Rearranging the above equ.

\[
V_s = \left( \frac{y}{2} \right) Z + 1 V_r + Z \left[ \frac{y}{4} Z + 1 \right] I_r
\]  
\( \text{(10)} \)

Now the sending end current is,

\[
I_s = Y V_m + I_r
\]  
\( \text{(11)} \)

Substituting the value of \( V_m \) to equation \( (11) \) we get,

\[
I_s = \frac{yV_r + [(y/2) Z + 1] I_r}{\Omega}
\]  
\( \text{(12)} \)

Again comparing equation \( (10) \) and \( (11) \) with the standard ABCD parameter equations,

\[
\begin{align*}
\frac{V_s}{I_s} &= A V_r + B I_r \\
\frac{V_r}{I_r} &= C V_r + D I_r
\end{align*}
\]

The parameters of the T network of a medium transmission line are

\[
\begin{align*}
A &= \left( \frac{y}{2} \right) Z + 1 \\
B &= Z \left[ \left( \frac{Y}{4} \right) Z + 1 \right] \Omega \\
C &= y \Omega \\
D &= \left( \frac{y}{2} \right) Z + 1
\end{align*}
\]

C. End Condenser Method

In this method, the capacitance of the line is limped or concentrated at the receiving or load end. This method of localizing the line capacitance at the load end overestimates the effects of capacitance.

\[ \]
\[
\begin{align*}
V_{\text{drop per phase}} &= \vec{I}_s \vec{Z} = \vec{I}_s (R + jX) \\
V_s &= \vec{V}_r + \vec{I}_s \vec{Z} = \vec{V}_r + \vec{I}_s (R + jX)
\end{align*}
\]

Now, 

\[
\% V_r = \left( \frac{V_s - V_r}{V_r} \right) \times 100\%
\]

and

\[
\% \varphi = \frac{\text{Power delivered / phase}}{\text{Power delivered / phase + losses / phase}} \times 100
\]

\[
= \frac{V_r I_r \cos \vartheta}{V_r I_r \cos \vartheta + I_s \cdot R} \times 100
\]

Limitations …… Although end condenser method for the solution of medium transmission line is simple to work out calculations, yet it has the following drawback:

a. There is a considerable error (about 10%) in calculations because the distributed capacitance has been assumed to be lumped or concentrated.

b. This method overestimates the effect of line capacitance

IV. EXAMPLE AND RESULT

Three phase transmission line have: (50 MW, 0.866 lagging p.f. and 132 KV) in receiving end.

If the line have (Z = 100 <80 Ohm / phase and Y = 0.0012 <90 S).

Calculate: V\_s, I\_s and power factor in sending end. Also find the transmission efficiency?

Enjoy the line is: short... medium.

Solution: Enjoy the voltage at the receiving end is reference and solve this problem as above expression and the results as follows:

<table>
<thead>
<tr>
<th>TABLE I.</th>
<th>RESULTS OF EXAMPLE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>V_s</td>
</tr>
<tr>
<td>Short line</td>
<td>163.5 KV</td>
</tr>
<tr>
<td>End capacity</td>
<td>149 KV</td>
</tr>
<tr>
<td>Π-method</td>
<td>156.2 KV</td>
</tr>
<tr>
<td>T-method</td>
<td>155.1 KV</td>
</tr>
</tbody>
</table>

V. CONCLUSION

Can see from the example, the line voltage is 132 KV, if used the short transmission line method to solve this example is cause large error in results because negligible the charging current. And see the the two method (Π and T) give approximate results and between the results of short transmission line method and capacity method.

REFERENCES